

Effective couplings and Perturbative Unification

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Abstract

In this paper we study the influence of the threshold effects due to massive degrees of freedom in the evolution with scale of gauge coupling constants. We first describe in detail the (standard) mass dependent renormalization prescription we use. This guides us to introduce and work with effective couplings, which are finite, process independent, and include complete threshold effects. We compute the evolution of the effective couplings in both, the Standard Model and its Minimal Supersymmetric extension, from m_Z to the high energy region. We find that the effects from thresholds due to the standard massive gauge bosons are non-negligible, contrary to what is generally assumed when using other, less-accurate, descriptions of the thresholds, as for example in the step-function approximation. Moreover, we find that thresholds are relevant when studying perturbative SUSY unification, changing the conclusions reached when using the step-function approach. We find that threshold effects bring conflict between the known experimental data at m_Z , the naturalness upper bound on the masses of SUSY partners and the *perturbative* unification of couplings.

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I. INTRODUCTION.

In studying Grand Unification theories for the strong, weak and electromagnetic interactions, we are compelled to work with two different scales: the electroweak symmetry breaking scale ($O(m_Z)$), on one hand, and the scale of the symmetry breaking of the unifying group G , or unification scale ($M_X \gtrsim 10^{16} \text{ GeV}$), on the other. In general, the unification schemes assume the existence of a “desert” between these scales (separated, at least, by 13 orders of magnitude), so that no new effects are present, beyond those coming from the unification group G .

To test the validity of the unification picture, one studies the evolution of the three gauge couplings of the Standard Model, from low (m_Z) to high energies (M_X), seeking for their convergence towards a common value at and beyond the unification scale M_X . In the simplest scenario, we can neglect the effects of the heavy degrees of freedom introduced by G in the evolution of the running couplings, and work only with the content of matter of the low energy theory (light degrees of freedom). This would be a first step in towards checking unification for the model. If, in this approximation, the couplings meet at a common value, g , at some point of the high energy region, M_X , we will have given the first step in the right direction.

The following step would consist of including the effects of the heavy degrees of freedom in our scheme. At low energy, these degrees of freedom are decoupled from the theory, but they become operative as soon as we approach the scale M_X , changing smoothly the evolution of the couplings. To take into account their effects in a proper way, one can integrate out the heavy fields from the complete action of the full theory, $S[G]$ [1,2]. Carrying this out, one obtains as unification condition: $g_i^{-2}(m_j^2 = g^{-2}(\mu) + \lambda_i(\mu, M_j))$, where the functions $\lambda_i(\mu, M_j)$ depend logarithmically on the masses of the heavy fields $M_j \approx O(M_X)$, the scale μ satisfies the condition $m_i \ll \mu \ll M_j$, with m_i the masses of the light fields. With this construction, the behavior of $g_i(\mu)$ is only dictated by the light degrees of freedom.

The main tool to carry out this two-step program are the renormalization group equations (RGE) and in particular the β -functions for the coupling constants, defined as $\beta_i = \frac{dg_i(\mu)}{d \ln \mu}$. In most of the Grand Unification analyses carried out, the β -functions used are those computed using the modified minimal subtraction (\overline{MS}) scheme [3]. But, since there is no unique renormalization prescription to work with, and different prescriptions yield different explicit forms of the β -functions, the question that naturally arises is *to which extent the results on unification depend on the prescription chosen*, that is, on the way one studies the evolution of the coupling constants with scale.

The choice of renormalization prescription is not a matter of taste. It implies the way one deals with physical effects, such as threshold effects, coming from the presence of massive particles in the theory. The \overline{MS} procedure is only one of the possible choices of a generic class of subtraction procedures, called mass independent subtraction procedures (MISP) [4]. For this class, the β -functions depend only on the particle *content* at the energy scale at which one computes them, and not explicitly on the masses of the particles. On the other hand, there also is available another generic class of procedures, called mass dependent subtraction procedures (MDSP) [5], which take into account the dependence *not only* on the number, *but also* on the particle *masses*. While with the resulting MISP- β -functions one is forced

to put in by hand the information about the mass spectrum of the theory when integrating the RGEs, with MDSPs one includes, in a natural way, all the information about threshold effects and possible decoupling of the massive particles in the *evolution* of the coupling constant [6]. Because of this feature, the MDSPs are therefore more complete and precise than the MISPs. They have the disadvantage that the calculation of the β -functions in MDSPs is far more complicated from a technical point of view.

Working with the \overline{MS} β -functions, the standard procedure to take into account the contribution of a particle of a given mass m at same scale μ , consists in making use of a “step-function” $\theta(\mu^2 - m^2)$: one for each massive particle [7]. This method constrains the contribution of a particle strictly to scales higher than its mass, being zero otherwise, and gives only the dominant logarithmic contribution of the mass. On the contrary, in the MDSP- β -function, the contribution of each massive degree of freedom is controlled via a *smooth* function $f(m/\mu)$, calculated in perturbation theory, that has the limits $f(m/\mu) \rightarrow 1$ when the ratio (m/μ) goes to zero, and $f(m/\mu) \rightarrow 0$ when (m/μ) goes to infinity. They give a non-zero contribution even for scales $\mu \leq m$, and a more accurate description of the threshold crossing than the step-function approximation (see for example, Fig. (10)). This behavior is a quantum-mechanical effect, reflecting the Heisenberg uncertainty principle: we have a non-zero probability of producing a particle even for momenta below the mass scale of the particle; furthermore, the contribution of the degree of freedom to the β -function spreads over a few orders of magnitude in momentum.

On the different treatment of threshold crossing resides the main source for the discrepancies in the final results derived with one or another approach. The change in the derivative of the coupling constant when crossing the threshold will affect the value of the couplings even when we are away from the threshold region at several orders of magnitude above the particle mass scale. And this will have clear implications in the study of unification theories with a rich low energy mass spectrum, such as for the Minimal Supersymmetric extension of the Standard Model (MSSM).

Previous work on the unification of the minimal susy model indicates that unification of gauge couplings is possible, and compatible with a susy spectrum below 1 *TeV* and a unification scale of $O(10^{16} \text{ GeV})$ [8,9]. These results were carried out by using the \overline{MS} β -functions, corrected with step-functions to treat the susy spectrum. Here, we will instead work with a different renormalization prescription, a mass dependent prescription, to include threshold effects. We will compare the results with those obtained by other authors. In this way, we will try to understand where and why are the differences between the conclusions reached with different approaches.

With the derivation we use to get threshold effects, we aim to clarify the point that these effects are not higher order corrections with respect the order of perturbation theory one is working, but instead they have to be calculated at *each* order of perturbation theory.

Before dealing with the susy model, we need, of course, to fix our prescription. We would have to choose one of the possible MDSPs. However, better than pick up a particular procedure and compute the β -function, we will see that the use of an MDSP allows us to define effective couplings, analogous to the effective charges defined by Gell-Mann and Low, Stueckelberg and Peterman in QED [10], in the sense that they are finite, universal, and include complete threshold effects, being by the way appropriate for our purposes. This will be the subject of Section 2. We will end that section with the definition of gauge effective

couplings associated with the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. Some problems related with the definition of the effective couplings for non-abelian theories, and also with broken symmetries, are also discussed in Section 2.

In Section 3 we first study the evolution of the gauge couplings of the SM with minimal matter contents: three generations of fermions, and one doublet of scalars. In this case, the number of massive degrees of freedom in the range (m_Z, M_X) is small, and we do not expect the unification of couplings to take place. In spite of this, the model becomes useful to point out the importance of threshold effects associated with massive gauge bosons, W^\pm and Z^0 , otherwise neglected in the step-function approximation.

We should keep in mind these contributions when studying the minimal susy extension of the SM, where more degrees of freedom (susy partners) are present beyond the scale m_Z . We will see in Section 4 that gauge boson threshold effects become relevant in attaining or not the “unification” of the gauge couplings. We first examine the model in the simplest approach (one-loop without unification gauge group thresholds); after that we will consider corrections due to heavy threshold effects in the standard way described in the beginning of this Introduction, and also will discuss the effects due to two-loop corrections.

In Section 5 we offer our conclusions, and discuss other approaches that have emerged in the recent literature [11–13]. The works cited on Ref. [13] remark the importance of complete light (susy) threshold effects for the determination of the \overline{MS} values of the gauge couplings at the electroweak scale, as they are extracted from experimental quantities, and the later implication of these initial values for unification. Susy thresholds distinguish between the quantities extracted using the SM, from the same quantities obtained on the MSSM. The main point of these papers is that not only the leading (logarithmic) threshold contributions are important, but also the non-leading, non logarithmic terms. In some sense, that is also our main statement. But we remark that here we study the impact of thresholds in the *evolution* with scale. Since we aim at comparing different evolutions, as given by two different approaches for the thresholds, we will assume the same initial conditions for the couplings in both schemes¹.

II. EFFECTIVE COUPLINGS.

Our purpose in this Section is to define effective couplings, following the simplest possible arguments from the general theory of renormalization. This is an easy task when the coupling is the coupling constant for an abelian gauge theory, as in QED. We will use what we learn in this case as a guide to get at the end the expression for non-abelian effective couplings.

For a general theory, the renormalized coupling, g_r , is related to the bare coupling, g_0 , by $g_r = Zg_0$, where Z is a product of renormalization constants: $Z = Z_3^{1/2}(Z_2Z_1^{-1})_a$. Here, Z_3 is the wave function renormalization constant (WFRC) of the gauge boson which defines the interaction; Z_2 is the product of WFRC of the external legs; and Z_1 is the proper vertex renormalization constant. The subindex “ a ” refers to the different classes of vertices

¹Further work in the determination of the initial values for the effective couplings at the electroweak scale in our scheme, is now in progress.

involving the same coupling constant, which come from the interaction of the gauge boson with itself or with the other particles present in the model: fermions, scalars, When using a MISP, the Slavnov–Taylor identities (Ward–Takahashi for abelian theories) guarantee that $(Z_2 Z_1^{-1})_a$ is independent of vertex we choose to define the coupling, and thus we obtain in this case a *universal* renormalized coupling, albeit without including threshold effects. With a MDSP, we include the dependence on the masses, in particular on the masses of the external legs, and we could, in principle, distinguish among the renormalized couplings derived from different processes: we would get a mass-dependent coupling, but *no a universal* coupling [14]. This would not be inconsistent with the constraints imposed by the Slavnov–Taylor identities. For example, one can define the renormalization constants associated with a specific process, and calculate the β -function, and after make use of the Slavnov–Taylor identities to derive the remaining renormalization constants for the theory, without modifying the previously defined β -function². But the renormalized coupling would not be universal (independent of the process). Therefore, in order to get a universal renormalized coupling, we first have to make sure that the MDSP respects the Slavnov–Taylor identities, using them to select the finite contributions (which contain the threshold effects) to the Z_i . This will guarantee the gauge invariance of our effective couplings.

For abelian gauge theories, such as QED, one can extend the Ward–Takahashi identity, $Z_2 = Z_1$, to the finite terms of the Z_i , and therefore the relation $e_r = Z_3^{1/2} g_0$ is valid for mass independent and mass dependent procedures. Moreover, we can see that, if we choose a suitable mass dependent procedure, the renormalized charge is equivalent to the effective charge of Gell–Mann and Low, Stuckelberg and Peterman, defined by:

$$e_{eff}^2(q^2) = \frac{e_0^2}{1 + \Pi_0^T(q^2)}. \quad (2.1)$$

Here, $\Pi_0^T(q^2)$ is the *transverse* component of the bare vacuum polarization tensor of the gauge boson. The effective charge is (a) independent of the renormalization scheme, since it is defined via bare functions, (b) universal and gauge independent and, (c) finite.

The last point is easily demonstrated when one replaces the bare coupling by the renormalized coupling in (2.1), since Z_3 must verify that the product $Z_3(1 + \Pi_0^T(q^2))$ be a finite function of both the scale q and the subtraction point μ . We therefore have:

$$e_{eff}^2(q^2) = \frac{e_r^2(\mu)}{(1 + \Pi_r^T(q^2, \mu^2))}, \quad (2.2)$$

where the explicit form of $\Pi_r^T(q^2, \mu^2)$ will depend on the chosen subtraction procedure.

In general, the two couplings e_{eff}^2 and e_r^2 are not equivalent. For example, the renormalized charge using \overline{MS} does not include threshold effects due to massive fermions, whereas they are present in e_{eff}^2 through the functions $\Pi_r^T(q^2, \mu^2)$. On the contrary, if we adopt a mass dependent procedure with the normalization condition $1 + \Pi_r^T(\mu^2, \mu^2) = 1$, then $e_{eff}^2(\mu^2) = e_r^2(\mu^2)$, and

$$\frac{\beta(e_{eff}^2)}{e_{eff}^2} = \frac{\beta(e_r^2)}{e_r^2}.$$

²This was explicitly shown for QCD in Ref. [15].

Therefore, both constants depend on the scale in the same way, and they include the threshold effects.

In the case of non-abelian theories, to get a finite expression for the effective couplings it is necessary something beyond the correction to the vacuum polarization. We must supply another function, $\Gamma_0(q^2)$, to the definition (2.1),

$$g_{eff}^2(q^2) = \frac{g_0^2}{(1 + \Pi_0^T(q^2, \mu^2))(1 + 2\Gamma_0(q^2))} = \frac{g_r^2(\mu)}{Z_3(1 + \Pi_0^T(q^2, \mu^2))(Z_2 Z_1^{-1})_a^2(1 + 2\Gamma_0(q^2))}, \quad (2.3)$$

which comes from the corrections due to vertex and external legs, and which must verify that the product $(Z_2 Z_1^{-1})_a(1 + \Gamma_0(q^2))$ be a finite function. The function $\Gamma_0(q^2)$ consists of a divergent term, Γ_0^{div} , and a finite contribution which includes a dependence on the masses. The Slavnov–Taylor identities guarantee that the divergent term of $(Z_2 Z_1^{-1})_a$ is the same for all possible vertices “ a ”, and is given by [16]:

$$(Z_2 Z_1^{-1})_a^{div} = 1 + \frac{g^2}{(4\pi)^2} C_2(G) \frac{3 + \xi}{4} \left(\frac{2}{n - 4} \right), \quad (2.4)$$

where $C_2(G)$ is the quadratic Casimir of the non-abelian group G , ξ is the gauge parameter, and n is the dimension of the space-time. Thus, $1 - \Gamma_0^{div} = (Z_2 Z_1^{-1})_a^{div}$.

But, it is not straightforward to calculate the finite contributions to $\Gamma_0(q^2)$, *i.e.*, to define $(Z_2 Z_1^{-1})_a$, with a mass-dependent procedure. The problem is to make sure that the resulting $\Gamma_0(q^2)$ is independent of the vertex “ a ” chosen to perform the calculation. For example, if we choose the vertex with fermions on the external legs, we can expect that $(Z_2 Z_1^{-1})_f$ depends on the fermion masses, the scalar masses, and the gauge boson masses (if the symmetry is broken). The same kind of masses may appear if we consider the scalar–gauge boson vertex, or the trilinear boson vertex. However, if we choose the ghost–gauge boson vertex, we only have running in the loop, ghosts, gauge bosons or Goldstone bosons (if the symmetry is broken): either massless particles, for unbroken symmetry, or particles with masses proportional to the gauge boson mass, for broken symmetry. Now, if we impose the Slavnov–Taylor identities, and thus $(Z_2 Z_1^{-1})_f = (Z_2 Z_1^{-1})_{ghost} = \dots$, it is clear that the *universal* correction $\Gamma_0(q^2)$ can not depend on the fermion or scalar masses, but only *on the gauge boson masses*.

In this sense, the universal threshold effects associated with the vertex are related to processes leading to gauge bosons production. Therefore, if we have an unbroken symmetry, there are no threshold effects from the vertex in the effective couplings, because we can always produce a massless particle. The corrections due to the presence in the vertex of other massive particles, such as fermions or scalars, will affect other parameters of the theory, but not the effective couplings. Moreover, these process depending corrections are finite, as it is implied by the Slavnov–Taylor identities.

So far, we have discussed about the general expressions of the effective couplings (for a general non abelian theory) given in terms of the functions $\Pi_0^T(q^2)$ and $\Gamma_0(q^2)$. The transverse bare vacuum polarization is easily calculated from the appropriate Feynman diagrams. We know the divergent part and the masses presents in the finite term of $\Gamma_0(q^2)$, but the explicit form of $\Gamma_0(q^2)$ will be closely related to the specific gauge theory we treat.

In particular, we are interested in studying the evolution of the gauge couplings of the Standard Model, g_3 , g and g' of $SU(3)_c \times SU(2)_L \times U(1)_Y$. We have no problems with the definition of $g_3(q^2)$, since QCD is an exact non abelian theory, so that:

$$g_3(q^2) = \frac{g_3^2}{1 + \Pi_{gg}^T(q^2) + 2\Gamma_3(q^2)}, \quad (2.5)$$

where,

$$\Gamma_3(q^2) = -g_3^2 \frac{3}{4} (3 + \xi) \left(\frac{2}{n-4} - \ln \frac{q^2}{\mu^2} + Constant \right), \quad (2.6)$$

and $\Pi_{gg}^T(q^2)$ is the transverse bare vacuum polarization of the gluon, given by the diagrams of Fig. (1).

To define the other two effective couplings, g and g' , we have to take into account that $SU(2)_L \times U(1)_Y$ is a broken symmetry at low energy (electroweak scale), just where we begin to run the couplings. In the broken phase, we have the three gauge bosons (the eigenstates of the mass matrix) W^\pm , Z^0 and the photon, A . We can define the effective coupling $g^2(q^2)$ by the interaction of the W^\pm ; however, since in this phase the gauge boson B , associated with the gauge symmetry $U(1)_Y$, is a mixed state of the neutral gauge bosons, it is better to work with the electromagnetic coupling, $e^2(q^2)$, given by the interaction of the photon, and define g' through the equation:

$$\frac{1}{g'^2} = \frac{1}{e^2} - \frac{1}{g^2}. \quad (2.7)$$

In the Standard Model, $e^2(q^2)$ is not a pure abelian coupling, so that to define $g(q^2)$ and also $e^2(q^2)$ we need the correction $\Gamma(q^2, m^2)$, where now “ m ” may be m_Z or m_W . This vertex correction is the same for both couplings, because the non abelian character of e^2 is closely related to g^2 ($e^{-2} = g^{-2} + g'^{-2}$). To obtain its explicit form we follow the argument given by Kennedy and Lynn [17], and that we partially reproduced in the following.

These authors make use of the relation between the non-abelian vertex correction, $\Gamma(q^2)$, and the longitudinal term of the mixed vacuum polarization tensor for the neutral bosons, $\Pi_{ZA}^L(q^2)$ [18]. The latter contribution gives rise to a non-diagonal mass matrix for the (Z^0, A) system. Therefore, one needs to redefine the fields Z^0 and A , which are the correct mass eigenstates at tree-level, in order to eliminate the non-diagonal term, and get the correct eigenstates (and a massless photon!) at one-loop order.

This can be carried out by first redefining the coupling g , including the universal vertex correction Γ with $\tilde{g} = g(1 - \Gamma)$, and then defining new fields \tilde{Z} and \tilde{A} making use of this coupling. Now, in this basis the non-diagonal term of the mass matrix becomes $\Pi_{ZA}^L + m_Z^2 g g' \Gamma / (g^2 + g'^2)$. Thus, if we choose Γ to satisfy the condition

$$\Pi_{ZA}^L + m_Z^2 \frac{g g'}{g^2 + g'^2} \Gamma = 0, \quad (2.8)$$

we get the desired results that: (a) the photon remains massless (at least at one-loop), and (b) we determine the explicit form of $\Gamma(q^2)$. Obviously, condition (2.8) is an identity for the

divergent terms of Π_{ZA}^L and Γ . What we get with (2.8) is the finite and mass-dependent term of Γ .

Now, we have all the ingredients involved in the definitions of $g^2(q^2)$ and $e^2(q^2)$, which are given by:

$$g^2(q^2) = \frac{g^2}{1 + \Pi_{WW}^T(q^2) + 2\Gamma(q^2)}, \quad e^2(q^2) = \frac{e^2}{1 + \Pi_{AA}^T(q^2) + 2\frac{e^2}{g^2}\Gamma(q^2)}, \quad (2.9)$$

where Π_{WW}^T , Π_{AA}^T are the transverse bare vacuum polarization tensor of the W^\pm and the photon respectively.

In Appendix A we give the expressions of the functions Π_{WW}^T , Π_{AA}^T and $\Gamma(q^2)$, as well as Π_{gg}^T . All these functions, including $\Gamma_3(q^2)$, depend on the gauge parameter ξ . This dependence cancels exactly for the divergent part of the combinations $\Pi_{ii}^T + \Gamma_i$, but the same does not happen for the finite term due to the presence of degrees of freedom whose masses are proportional to ξ (i.e., gauge bosons, ghosts, and except in Π_{gg}^T and $\Gamma_3(q^2)$, also Goldstone bosons). This leads to effective couplings depending on the gauge parameter [15].

The gauge parameter also changes with scale, as given by its RGE

$$\frac{d\xi}{d \ln q^2} = -\xi \frac{d \ln Z_3}{d \ln q^2}, \quad (2.10)$$

and it will depend on the associated³ coupling $g_i^2(q^2)$ through the dependence of Z_3 . In order to get the evolution of the effective couplings we have to solve a system of coupled differential equations. The change in $g_i^2(q^2)$ produced by the change in $\xi(q^2)$ will be small (order two-loop); and if we calculate the effective coupling at one-loop order, we can neglect it, and maintain $\xi(q^2)$ at its initial value. But if we calculate $g_i^2(q^2)$ at two-loop order we need, at least, $\xi(q^2)$ at one-loop order, and so on. On the other hand, the theory by itself indicates the most suitable value of ξ to work with, without approximations: the fixed point of the differential equation at $\xi = 0$. Thus, we choose to work in the Landau gauge.

Up to now, we have defined the effective couplings in the Standard Model, which are *finite, universal and include threshold effects*. For the $SU(2)_L \times U(1)_Y$ couplings, we made partially use of the arguments given by Kennedy and Lynn to derive the function $\Gamma(q^2)$. Nevertheless, our effective couplings do not coincide exactly with their definitions. In particular, we differ in the definition adopted for $g(q^2)$. Here, we have chosen to relate this coupling directly to the W^\pm propagator, in the same way that the coupling $e(q^2)$ is related to the photon propagator. They choose, instead, the mixed propagator of the photon and the W^3 boson. Their convention follows from imposing that the tree level relation among e^2 , g^2 and the sine of the mixing angle, $\sin^2 \theta_W = s_W^2$, had to be maintained at the level of the effective parameters, *i.e.*,

$$\frac{s_W^2(q^2)g_{KL}^2(q^2)}{e^2(q^2)} = 1, \quad (2.11)$$

³We will have four different gauge parameters, associated with the bosons W^\pm , A and Z^0 and gluons respectively.

where $s_W^2(q^2)$ is derived from neutral current processes.

In our case, to get a definition of $s_W^2(q^2)$ consistent with the neutral current amplitude, we have to allow for the relation (2.11) to receive radiative corrections [19], so that,

$$\frac{s_W^2(q^2)g^2(q^2)}{e^2(q^2)} = 1 + O(\hbar). \quad (2.12)$$

The coupling $g_{KL}^2(q^2)$ is related to a conserved current, and hence it only receives contributions from loops of charged and degenerate particles. There is, for example, no contributions due to the Higgs. On the other hand, all the doublets under $SU(2)_L$ contribute to the coupling $g^2(q^2)$, and this includes the Higgs. The coupling $g^2(q^2)$ is related with a *broken* symmetry, and this fact is not reflected in $g_{KL}^2(q^2)$. *We consider that the definition of $g^2(q^2)$ is more appropriate for our purposes, whereas $g_{KL}^2(q^2)$ is more appropriate to study the neutral current and related processes.*

In the following sections, we will proceed to study the evolution with scale of the effective couplings from m_Z to the high energy region, where we want to check for the validity of the unification scenario. With this in mind, we see that the above definitions of the effective couplings will be useful once we eliminate the bare couplings in favor of the couplings given at the scale m_Z . Furthermore, bearing in mind that the combinations $\Pi_{ii}^T + 2\Gamma_i$ are proportional to g_i^2 , we redefine $\Pi_{ii}^T + 2\Gamma_i = g_i^2(\Pi_{ii}^T + 2\Gamma_i)$, and obtain:

$$\frac{1}{g_i^2(q^2)} = \frac{1}{g_i^2(m_Z^2)} + \left(\Pi_{ii}^T(q^2) + 2\Gamma_i(q^2) - \Pi_{ii}^T(m_Z^2) - 2\Gamma_i(m_Z^2) \right). \quad (2.13)$$

As an example of unifying group we will take the minimal choice, i.e., $G = SU(5)$.

III. THRESHOLD EFFECTS IN THE STANDARD MODEL.

In this section we will study the Standard Model with minimal matter contents, i.e., three generations of fermions and one scalar doublet. From the point of view of unification, we do not expect that the threshold effects included in the definition of the effective couplings can change the negative results obtained with other renormalization procedures [8]. This model is simply intended to see how thresholds affect the evolution of gauge couplings [20].

Since for scales $\mu \geq m_Z$ we can regard all the fermions as being massless, except for the top quark, the relevant masses for the problem are m_Z , m_W , m_t (top) and m_h (Higgs). The values of m_Z and m_W are determined by experiments to be at [21]: $m_Z = 91.187 \pm 0.007 \text{ GeV}$, $m_W = 80.2 \pm 0.3 \text{ GeV}$. But m_t and m_h remain as free parameters, with lower experimental bounds⁴: $m_t \geq 131 \text{ GeV}$, $m_h \geq 60 \text{ GeV}$ [23]. On the other hand, imposing as condition the validity of perturbation theory in the range of scales considered, one gets the upper

⁴ The recent data from the CDF Collaboration at Fermilab [22] indicate the existence of the top quark with $m_t = 174 \pm 10_{-12}^{+13} \text{ GeV}$. However, this data is still to be confirmed, and therefore we would maintain the lower bound given in Ref. [21]. In any way, to fix or let the top mass free makes only some very tiny differences in the numerical results, and it does not influence the conclusions reached in this paper.

theoretical limit: $m_t, m_h \leq 200 \text{ GeV}$ [24]. We will use in this section these latter values for m_t and m_h .

The initial values of the couplings, $g_i^2(m_Z)$, are identified with the experimental values measured at the scale m_Z :

$$\alpha_3(m_Z) = 0.117 \pm 0.005, \quad [21] \quad (3.1)$$

$$\alpha_e^{-1}(m_Z) = 127.9 \pm 0.3 \quad [25] \quad (3.2)$$

$$\sin^2 \theta_W(m_Z) = 0.2319 \pm 0.0005 \quad [21]. \quad (3.3)$$

In what follows, we will work with the “constants” $\alpha_i = g_i^2/4\pi$. The initial value of α_2^{-1} is fixed by the relationship: $\alpha_2^{-1}(m_Z) = \alpha_e^{-1}(m_Z) \sin^2 \theta_W(m_Z)$. And, as mentioned above, the evolution of α_1^{-1} is calculated by the relation $\alpha_1^{-1} = 3(\alpha_e^{-1} - \alpha_2^{-1})/5$, where the normalization factor is due to the embedding of $U(1)_Y$ in $SU(5)$.

In Fig. (2) we have plotted the effective couplings ($\alpha_i^{-1}|_{ef}$) in the high energy region. We have also plotted the couplings calculated in the step-function approximation ($\alpha_i^{-1}|_\theta$), and using minimal subtraction ($\alpha_i^{-1}|_{MS}$). The latter procedure is equivalent to the step-function approximation when all masses are below or at the scale m_Z . We see in the plot that the approximation we use for α_1^{-1} and α_3^{-1} makes only very little difference. Nevertheless, we can see that, due to threshold effects produced by the top, $\alpha_3^{-1}|_{ef}$ is a little larger than $\alpha_3^{-1}|_\theta$, and this, in turn, a little larger than $\alpha_3^{-1}|_{MS}$. The top quark decouples at low energies ($m_t \ll 10^{12} \text{ GeV}$), but its effect propagates to the high energy region by the RGE, being the decoupling in $\alpha_3^{-1}|_{ef}$ smoother than in the θ -approximation.

For the coupling α_2^{-1} , one would naively have expected that the threshold contributions due to m_t and m_h contribute to increase its slope. But these are not enough, because in this case the dominant threshold effects are those due to the massive gauge bosons, so that the slope of $\alpha_2^{-1}|_{ef}$ decreases, as can be seen from the plot. This effect is not present in $\alpha_2^{-1}|_\theta$ (and $\alpha_2^{-1}|_{MS}$) because we begin to integrate *precisely* at the scale of m_Z . This can be seen clearly if we compare the derivatives, given by:

$$(4\pi) \frac{\partial \alpha_2^{-1}|_\theta}{\partial \ln \mu^2} = \frac{22}{3} \theta(\mu^2 - m_Z^2) - \frac{1}{12} \theta(\mu^2 - m_h^2) - \theta(\mu^2 - m_t^2) - \frac{37}{12}, \quad (3.4)$$

$$(4\pi) \frac{\partial \alpha_2^{-1}|_{ef}}{\partial \ln \mu^2} = \frac{13}{3} (s_W^2 f_g(0, a_W) + c_W^2 f_g(a_Z, a_W)) + 3f_\Gamma(a_W, a_W) - \frac{1}{12} f_s(0, a_h) - f_f(0, a_f) - \frac{37}{12}, \quad (3.5)$$

where the functions $f_b(a_i, a_j)$ are defined in Appendix A, and smooth out the “step” of the Heaviside function, as demanded by the Heisenberg uncertainty principle (see Fig. 3). The derivative of $\alpha_2^{-1}|_\theta$ changes from negative to positive for $q^2 = m_Z^2$, while in the case of $\alpha_2^{-1}|_{ef}$ threshold effects postpone this change of sign [14]. Therefore, $\alpha_2^{-1}|_{ef}$ decreases when we begin the integration, and this makes the differences with $\alpha_2^{-1}|_\theta$ sizable only in the high energy region.

The differences between step-function and effective couplings are in that for the former, thresholds are considered as an “instantaneous” effect; we “switch on” and “switch off” them at a specific point of the energy scale, i. e., at $q^2 = m_i^2$. But indeed, thresholds are spread over a certain range in the momentum scale. The threshold crossing is not “instantaneous”.

As shown in Appendix A, only when we begin to integrate at a scale $q_0^2 \gg m_i^2$ (massless approximation), *or* we end the integration at $q^2 \ll m_i^2$ (complete decoupling), we get the same contribution for the massive degree of freedom m_i , in both $\alpha_i^{-1}|_\theta$ and $\alpha_i^{-1}|_{eff}$. In the general case, when $q_0^2 < m_i^2 < q^2$, even if we can apply the limit $m_i^2 \ll q^2$, and then suppress all the terms $O(m_i/q)$ in the threshold function contribution, we will get that,

$$\alpha_i^{-1}|_\theta \sim \ln \frac{m_i^2}{q^2},$$

while,

$$\alpha_i^{-1}|_{eff} \sim \ln \frac{m_i^2}{q^2} - \ln C_i,$$

where C_i is a “constant” of $O(10)$ ⁵. This constant contribution is not more than a reminder of the fact that, although if the particle is decoupled at q_0 , in going to q we have to cross the region of scales $O(m_i)$, where the degree of freedom is neither decoupled nor coupled⁶. Moreover, the transition between the decoupled–coupled regimes, takes at least more than one order of magnitude in the energy scale. Therefore, when beginning the integration at the electroweak scale, m_Z , it is not a good approximation to consider the massive gauge bosons as completely coupled.

We can now compare the corrections due to threshold effects with those coming from improving the order of perturbation theory in the mass-independent approach. In Fig. (4) we have plotted $\alpha_i^{-1}|_{ef}$ at one-loop order, and $\alpha_i^{-1}|_{MS}$ at two-loop order. We also include $\alpha_i^{-1}|_{MS}$ at one-loop order as a reference. We can now see that two-loop effects are qualitatively similar to threshold effects. Both of them raise α_3^{-1} and decrease α_2^{-1} , although for α_2^{-1} the thresholds from the gauge bosons dominate over two-loop effects. We can expect that, when we take into account both corrections, calculating effective couplings at two loop order, the differences relative to the values of $\alpha_i^{-1}|_{MS}$ become more accentuated.

Instead of doing the exact calculation, we have decided to study this case in an approximate way. We use it only as an indication of which kind of behavior we can expect at 2-loop order with thresholds. In Appendix B we describe in detail the argument followed to obtain approximated two-loop effective couplings, based on the expressions for the RGEs at 2-loops, and general properties of the threshold functions. When we study their evolution, we find what we expected: α_2^{-1} decreases and α_3^{-1} increases slightly, with respect to the one loop $\alpha_i^{-1}|_{ef}$ values. Of course, these effects are not enough to “unify” the three gauge couplings, but it is useful to keep them in mind when studying theories with a larger presence of massive degrees of freedom.

In spite of the no unification of the couplings, the Standard Model with minimal matter contents has been useful to study the threshold effects introduced in the effective couplings.

⁵“Constant” means that this term can be approximated by a constant. See Appendix A for details.

⁶The word “coupled” is used here as opposite to “decoupled”, i. e., maximum contribution to the β functions.

In particular, we have seen that qualitatively, the thresholds associated with the gauge bosons are not negligible, contrary to what is assumed by the step-function approximation (since we begin to integrate at the scale $q^2 = m_Z^2$). Quantitatively, moreover, this correction is larger than the two-loop correction.

It is clear that thresholds are additive and they will become more relevant the larger the number of degrees of freedom in the range $[m_Z, M_X]$. This is the case of the Minimal Supersymmetric extension of the Standard Model (MSSM), that we will study in the following section.

IV. UNIFICATION IN THE MSSM.

If supersymmetry is a symmetry of Nature, every particle of spin “ j ” of the Standard Model must come with its supersymmetric (susy) partner, of spin “ $j \pm 1/2$ ”. Thus, we have gauginos, sleptons, squarks and higgsinos in addition to the content of matter in the Standard Model. We also need, at least, two Higgs doublets (with their higgsinos) in order (a) to cancel the anomaly due to the higgsinos, and (b) to give mass to the two components of the quark doublet. Since the susy particles have not been detected, susy must be broken at low energies, and therefore we have a larger number of arbitrary parameters of masses to consider beyond the scale m_Z .

One can try to constraint the susy spectrum in the context of unification theories, making use not only of limits on proton decay [26,27], but also of cosmological arguments on the relic abundance of the lightest susy particle (LSP) [28]. In general, the susy spectrum thus obtained lies below 1 TeV , and thus it is therefore compatible with the stability of the hierarchy of scales in the model under radiative corrections. In this sense, the limit of “ 1 TeV ” is usually denoted as the “naturalness” bound.

Most of the predictions have been obtained making use of the step-function approximation to deal with susy thresholds. But the more accurate description of threshold effects, such as given in the effective couplings, could change the conclusions reached as we will show in this section [29].

Of the susy spectrum, we will assume that susy is broken at the Planck scale by a “hidden” sector, so that the large number of susy masses can be determined at the weak scale in terms of a small number of parameters given at the unification scale [30]. Accordingly, we take the following parametrization, neglecting the (higher order) mixing between winos and higgsinos, and the s-tops left and right for simplicity:

(a) For winos (\tilde{w}), and gluinos (\tilde{g}),

$$m_{\tilde{w}} = m_{1/2}, \quad m_{\tilde{g}} = 3m_{1/2},$$

(b) for squarks (\tilde{q}), and sleptons left (\tilde{l}_L) and right (\tilde{l}_R),

$$m_i^2 = m_{1/2}^2(c_i + \xi_0), \quad c_{\tilde{q}} = 7, \quad c_{\tilde{l}} = 0.5, \quad c_{\tilde{\tau}} = 0.15,$$

where $\xi_0 = (m_0/m_{1/2})^2$, m_0 being the common mass for the scalars, and $m_{1/2}$ being the gaugino mass at the unification scale; and

(c) for higgsinos, $m_{\tilde{h}}$ will be taken as arbitrary.

Furthermore, there are five massive fields coming from the two Higgs doublets: two charged Higgses (m_+), two neutral Higgses (m_h, m_H), and one pseudoscalar (m_a), whose masses must satisfy the relations: $m_+^2 = m_a^2 + m_W^2$, $m_H^2 = m_a^2 + m_Z^2 - m_h^2$. The lighter neutral Higgs is identified with the standard Higgs, and for the remaining we take $m_+^2 \approx m_a^2 \approx m_H^2$. At the end, we have as arbitrary mass parameters the following: $m_t, m_h, m_{1/2}, m_0, m_{\tilde{h}}$ and m_H . The value of $m_{1/2}$ is bounded from below by the experimental searches for charginos [21], $m_{1/2} \geq 45 \text{ GeV}$. We take the remaining susy parameters at least of order m_Z .

The expressions of the effective couplings given in Section III, with the functions $\Pi_{ii}^T + 2\Gamma_i$ calculated in Appendix A, do not include the contribution of the susy degrees of freedom. However, these are easily derived, taking into account that (a) they do not appear in Γ_i , but only in Π_{ii}^T , and (b) they are contributions from fermions or scalars (Eqs. (A11) and (A10)).

As a first attempt to look for the unification of the effective couplings, we simply study their evolution with scale, without making any reference to the unifying group. In Fig. (5.a) and (5.b) we have represented $\alpha_i^{-1}|_{ef}$ for two different values of the susy parameters (the lower and upper bound). In order to make a meaningful comparison, we have also included $\alpha_i^{-1}|_{\theta}$ calculated with the same susy parameters. Taking into account the experimental errors in the coupling constants, we see that the $\alpha_i^{-1}|_{\theta}$ unify (they *cut* at one point, M_X , of the energy scale), for both values of the susy parameters. Due mainly to the experimental error in $\alpha_3(m_Z)$, at this level of approximation one can not extract much more information about the susy spectrum with the $\alpha_i^{-1}|_{\theta}$.

But the situation is not as unambiguous for the $\alpha_i^{-1}|_{ef}$: to get unification, effective couplings *prefer* higher values for the susy parameters, around the naturalness bound of 1 TeV . Now, the threshold effects due to the gauge bosons play an important role in reaching or not unification. While the contribution of susy masses (fermions and scalars) tends to increase the slope of $\alpha_3^{-1}|_{ef}$ and $\alpha_1^{-1}|_{ef}$, for the coupling $\alpha_2^{-1}|_{ef}$ this effect is compensated by the decrease of the slope due to the gauge thresholds. Because of this, the differences between $\alpha_2^{-1}|_{ef}$ and $\alpha_2^{-1}|_{\theta}$ are almost negligible, contrary to those obtained for the other two couplings, making $\alpha_2^{-1}|_{ef}$ move away from the “crossing” point with the other two effective couplings. Indeed, if the bosons W^\pm and Z^0 were massless, the slope of $\alpha_2^{-1}|_{ef}$ would increase sufficiently so as to allow the unification of the three couplings even for a susy spectrum of $O(m_Z)$ (see Fig. (6)). *Qualitatively*, we will achieve the same conclusions as we would if using the step-function approximation: the perturbative unification of the couplings is compatible with a susy spectrum below 1 TeV , and a unification scale M_X high enough to fulfill the experimental lower limit on proton decay. However, when we take into account the thresholds, including those of W^\pm and Z^0 , this general conclusion is slightly modified, and we get more information about the susy spectrum: it can not be of $O(m_Z)$. We see that the electroweak breaking, being a “low energy” process (order m_Z), has effects on processes which take place at “high energy” (order of M_X), such as the perturbative unification of the couplings.

Simply plotting the evolution of the couplings is not the best way for making predictions, and even less so with the big uncertainty we have in the experimental determination of $\alpha_3(m_Z)$. Indeed, of the three gauge couplings the value of $\alpha_3(m_Z)$ is the worst known experimentally, in the sense that there is not a general agreement between different measurements [31–33]. In general, the values obtained from experiments at low energies (below m_Z) are lower than the values inferred from the decay of the Z^0 into hadronic states (LEP

data) (see Table I). Because of this discrepancy it is better not to take any value of $\alpha_3(m_Z)$ as the initial data and, instead, *to derive* it from the unification condition:

$$\alpha_1^{-1}(M_X) = \alpha_2^{-1}(M_X) = \alpha_3^{-1}(M_X) = \alpha_G^{-1}. \quad (4.1)$$

In this way, susy masses can be bounded by demanding that $0.108 \leq \alpha_3(m_Z) \leq 0.125$, which are the experimental lower and upper values quoted in Table I.

Solving the system of equations derived from (4.1), one gets the general expressions:

$$\alpha_3^{-1}(m_Z) |_{\theta} = \frac{1}{7}(15\alpha_2^{-1}(m_Z) - 3\alpha_e^{-1}(m_Z)) + \frac{1}{56\pi} \sum_i d_i f\left(\frac{m_i}{m_Z}\right), \quad (4.2)$$

$$\ln \frac{M_X}{m_Z} |_{\theta} = \frac{\pi}{14}(3\alpha_e^{-1}(m_Z) - 8\alpha_2^{-1}(m_Z)) - \frac{1}{84} \sum_i d'_i f\left(\frac{m_i}{m_Z}\right), \quad (4.3)$$

where the sums run over *all* the masses (susy and non-susy). When we work within the step-function approximation, the functions “ f ” are simply given by $\ln(m_i^2/m_Z^2)$; thus, only masses m_i greater than m_Z contribute. Within the effective couplings approach, and making use of the approximation for the threshold function given in Appendix A (Eq. (??)), we get

$$f(m_i/m_Z) = \ln \frac{m_Z^2 + c_i m_i^2}{m_Z^2}, \quad (4.4)$$

where the value of the c_i mostly depends on the kind of massive particle running in the loop (see Table II). As we have seen in Sect. 3, and also in Appendix A, this constant is intended to smooth the threshold-crossing, and to control, not only beyond but also *below*, how far it is the threshold from the electroweak scale.

Therefore, we see that, independently of whether we use effective couplings or the step-function approximation, the behavior with susy masses of $\alpha_3^{-1}(m_Z)$ and $\ln M_X$ will be the same: the higher the susy masses, the higher $\alpha_3^{-1}(m_Z)$ and the lower is $\ln M_X$. But now, with the effective couplings we have also a non zero contribution of the massive gauge bosons, which are explicitly given by:

$$\alpha_3^{-1}(m_Z) \rightarrow \frac{1}{28\pi} \left(39s_W^2 \ln \frac{m_Z^2 + 2m_W^2}{m_Z^2 + 5m_W^2} - 52 \ln \frac{m_Z^2 + 5m_W^2}{m_Z^2} - 36 \ln \frac{m_Z^2 + 2.5m_W^2}{m_Z^2} \right) \approx -1.44, \quad (4.5)$$

$$\ln M_X \rightarrow \frac{1}{168} \left(26s_W^2 \ln \frac{m_Z^2 + 2m_W^2}{m_Z^2 + 5m_W^2} + 65 \ln \frac{m_Z^2 + 5m_W^2}{m_Z^2} + 45 \ln \frac{m_Z^2 + 2.5m_W^2}{m_Z^2} \right) \approx +0.88. \quad (4.6)$$

Thus, these terms decrease the value of $\alpha_3^{-1}(m_Z)$ and increase the value of $\ln M_X$, with respect to those obtained in the step-function approximation.

This behavior is clearly seen when we compare Fig. (7) (effective couplings) and Fig. (8) (step-function), where we have plotted $\alpha_3^{-1}(m_Z)$ versus $\log_{10}(m_{1/2})$, for different values of the remaining mass parameters. The allowed region in the map $\alpha_3^{-1}(m_Z) - \log_{10} m_{1/2}$ is delimited by the experimental bounds on $\alpha_3^{-1}(m_Z)$ and $m_{1/2}$ given above, and also by the theoretical limit $M_X \geq 10^{16} \text{ GeV}$, which is a safe bound for proton decay. Since higgsinos and heavy Higgses contribute with the same sign, we have simply taken $m_{\tilde{h}} = m_H$. Moreover, $\alpha_3^{-1}(m_Z)$ and $\ln M_X$ depend only very slightly on the parameter $\xi_0 = (m_0/m_{1/2})^2$, so that we have taken $\xi_0 = 1$ as a typical value for the plot. As we increase the mass of the higgsinos, we

decrease (increase) the value of $\alpha_3(m_Z)$ ($\alpha_3^{-1}(m_Z)$) compatible with unification. However, since the step-function procedure does not distinguish masses lower than m_Z with *any* higgsino mass below m_Z and gauginos masses of $O(m_Z)$, we get the limit $\alpha_3(m_Z) < 0.121$, perfectly compatible with the experimental band for this coupling. As we have previously seen, no constraint on the susy spectrum is obtained at this order. On the other hand, with the effective couplings we would get the upper value of $\alpha_3(m_Z)$ in the limit $m_{\tilde{h}} \ll m_Z$, but, now, due to the contribution of the gauge bosons, this limit has no physical interest ($\alpha_3(m_Z) < 0.140$). The point is that also for higgsinos masses of $O(m_Z)$, and almost for a susy spectrum of $O(1\text{ TeV})$, the value we get for $\alpha_3(m_Z)$ is too high. Turning the argument upside down, if we demand $\alpha_3(m_Z) \leq 0.125$ and $M_X \geq 10^{16}\text{ GeV}$, we get a lower bound on $m_{\tilde{h}} = m_H$, and an upper bound on $m_{1/2}$. These bounds depend on m_t and m_h (the lower m_t and m_h , the higher $m_{1/2}$ and $m_{\tilde{h}}$), and also on ξ_0 , although these bounds remain practically unchanged for $\xi_0 \geq 10^4$. For this value of ξ_0 we get:

$$\begin{aligned} m_{1/2} &\leq 7\text{ TeV} \quad (m_h = 60\text{ GeV}, \quad m_t = 91\text{ GeV}), \\ m_{\tilde{h}} &\geq 370\text{ GeV} \quad (m_h = m_t = 200\text{ GeV}, \quad m_{\tilde{h}} = m_H). \end{aligned} \quad (4.7)$$

By giving these bounds and the above bounds on $\alpha_3(m_Z)$, we have taken into account the experimental errors in $\sin^2\theta_W(m_Z)$ and $\alpha_e^{-1}(m_Z)$. For the central values of these quantities we would obtain: $m_{1/2} \leq 3.8\text{ TeV}$, $m_{\tilde{h}} \geq 1\text{ TeV}$. Any kind of prediction from unification is very sensitive to small variations in the input parameters, and even small errors in the experimental data induce serious uncertainties⁷.

With this calculation we have recovered in a more detailed way the general results we obtained before. At the one-loop order, we can get the perturbative unification of the couplings, compatible with the naturalness bound for the susy masses and proton decay-lifetime limits, independently of whether we take or not threshold effects into consideration. But the inclusion of more accurate thresholds than those described by the step-function, drives the susy spectrum close to the 1 TeV bound and the value of $\alpha_3(m_Z)$ into the range of LEP data.

Threshold effects as considered here are not higher order corrections, but a correction to the simplest step-function approximation. As a first approach, just at one-loop order, this latter procedure gives a good indication for unification with a viable susy spectrum. Nevertheless, when one improves the treatment of thresholds, working with effective couplings, although the conclusions are not drastically changed, the result at one-loop does not leave much room for more corrections, for example 2-loop order corrections.

In fact, it is not necessary to carry out a complete calculation of thresholds at two-loop to get the general behavior. It is enough to take into account that two loop effects tend to lower the value of $\alpha_3^{-1}(m_Z)$ by about 10 %, for both the step-function and effective couplings⁸. In the first case, this effect drives the upper limit obtained at one-loop order

⁷ Note that the values quoted in [29] are different, due to the different value of $\sin^2\theta_W(m_Z)$ considered in that work.

⁸ For the effective couplings, this can be easily shown by resorting to the kind of approximation given in Appendix B.

towards the experimental upper limit ($\alpha_3(m_Z) \leq 0.125$), while the naturalness bound on susy masses gives now the lower bound $\alpha_3(m_Z) \geq 0.121$. With the effective couplings, a 10% decrease pushes the values of $\alpha_3^{-1}(m_Z)$ obtained with $m_{\tilde{h}} = m_H \leq 1 \text{ TeV}$ away from the experimental band. We would need masses of order 10 TeV to get $\alpha_3(m_Z) \leq 0.125$. For the two-loop calculation, the dependence on the parameter ξ_0 is not so mild, and the value of $\alpha_3^{-1}(m_Z)$ increases with it. In any case, to have $m_{\tilde{h}} = m_H$ in the range of TeV , we would need $\xi_0 \geq 10^{10}$, i.e., squarks and sleptons of order 1 PeV . Therefore, we can not conclude, at two-loop order, that the simple scenario for perturbative unification is *viable*; in the sense that it is not *compatible* with the experimental data on $\alpha_3(m_Z)$ and the naturalness bound on the susy spectrum.

The question still remains of the corrections due to the heavy degrees of freedom associated to the grand unification group (in our case $G = SU(5)$). As we have seen previously, light thresholds introduce non negligible differences in the evolution of the running couplings, and the same will take place with the heavy masses (M_j), mainly when the ratio μ/M_j begin to approach to one. The evolution of the effective couplings, including only the light degrees of freedom, is valid up to scales $\mu \ll M_j$, where we are sure about the decoupling of the heavy masses M_j . The relationship between the couplings $\alpha_i^{-1}(\mu)$ and $\alpha_G^{-1}(\mu)$ is obtained integrating out the heavy degrees of freedom from the complete action $S[G]$, and is given by:

$$\alpha_i^{-1}(\mu) = \alpha_G^{-1}(\mu) + \lambda_i(\mu, M_j). \quad (4.8)$$

The functions $\lambda_i(\mu, M_j)$ include logarithmic contributions of the heavy masses, as well as a constant term due to the light degrees [1]. As we point out in Sect. 3, the logarithmic contribution is what we get when applying the limit $M_j \gg \mu$ to the threshold functions.

For non-susy $SU(5)$, we have three basic parameters to describe the heavy masses: M_V (gauge bosons), M_Φ (colored Higgs) and M_Σ (heavy scalar in the adjoint representation). For susy $SU(5)$ we have to add the associated susy partners. If we assume, as it is common, that the members of each heavy supermultiplet are degenerated, we get⁹:

$$(4\pi)\lambda_1(\mu, M_j) = \frac{66}{5} + \frac{96}{5} \ln \frac{M_V}{\mu} - \frac{4}{5} \ln \frac{M_\Phi}{M_V} - \frac{20}{3} \ln \frac{M_\Sigma}{M_V}, \quad (4.9)$$

$$(4\pi)\lambda_2(\mu, M_j) = \frac{41}{6} + 8 \ln \frac{M_V}{\mu} - 8 \ln \frac{M_\Sigma}{M_V}, \quad (4.10)$$

$$(4\pi)\lambda_3(\mu, M_j) = \frac{5}{4} - 2 \ln \frac{M_\Phi}{M_V} - \frac{26}{3} \ln \frac{M_\Sigma}{M_V}. \quad (4.11)$$

These expressions are valid at one-loop order for scales μ such that $m_i \ll \mu \ll M_j$. Since

⁹The expressions given in Ref. [29] for $\lambda_3(\mu, M_j)$ and also $\ln M_V$ included minor mistakes that we have corrected here (see Eq. (4.11) and (4.13)). We have also corrected the constant term due to the light degrees of freedom that appears in λ_i . In Ref. [29] we reproduced the terms given by N.-P. Chang et. al. in [1]. Here, we have recalculated them to be compatible with our definition of effective couplings. Anyway, the numerical differences are small, and they do not affect the conclusions.

heavy masses are typically of order 10^{16} GeV , and light masses are expected to be less than 1 TeV , we choose $\mu = 10^7 \text{ GeV}$.

As before, from (4.8) we get a system of equations, where now the unknowns will be α_G^{-1} , $\alpha_3^{-1}(m_Z)$, and $\ln M_V$ instead of $\ln M_X$. We get for $\alpha_3^{-1}(m_Z)$ and $\ln M_V$ the following expressions:

$$\alpha_3^{-1}(m_Z) = \frac{1}{2} \left(3\alpha_2^{-1}(m_Z) - \alpha_1^{-1}(m_Z) \right) + \frac{1}{8\pi} \sum_i g_i F_i(m_i, \mu) - \frac{3}{5\pi} \ln \frac{M_\Phi}{\mu} - \frac{3}{5\pi}, \quad (4.12)$$

$$\ln M_V = \frac{3\pi}{8} \left(\alpha_1^{-1}(m_Z) - \alpha_2^{-1}(m_Z) \right) + \frac{3}{32} \sum_i g'_i F_i(m_i, \mu) + \frac{3}{40} \ln \frac{M_\Phi}{\mu} - \frac{1}{8} \ln \frac{M_\Sigma}{\mu} - \frac{191}{320}, \quad (4.13)$$

where, as before, the sums run over all the light masses. These expressions are valid for both effective couplings and step-function approximation. In the first case, the $F_i(m_i, \mu)$ are the approximated threshold functions defined in (A33); in the second case, we simply take $F_i(m_i, \mu) = \ln(m_i^2/\mu^2)$ when $m_i \geq m_Z$, and $F_i(m_i, \mu) = \ln(m_Z^2/\mu^2)$ when $m_i < m_Z$. Notice that for $\alpha_3^{-1}|_\theta$ and $\ln M_V|_\theta$, the dependence on the scale μ is exactly canceled out at one-loop order. For the effective couplings, this dependence remains negligible if we maintain the condition $m_i \ll \mu \ll M_j$.

The qualitative behavior of $\alpha_3^{-1}(m_Z)$ and $\ln M_V$ with susy masses is the same observed without taking into account the heavy thresholds.

Our interest now shifts to the heavy mass parameters M_Φ and M_Σ .

Since the value of $\alpha_3^{-1}(m_Z)$ only depends on M_Φ , the limits on this parameter will put bounds on $\alpha_3^{-1}(m_Z)$. The lower bound on M_Φ comes from the experimental limits on proton decay via dimension five operators. The lifetime for the dominant mode is given by [27]:

$$\tau(p \rightarrow K^+ \bar{\nu}_\mu) = 6.9 \times 10^{31} \left| \frac{0.003 \sin 2\beta_H}{\beta} \frac{M_\Phi}{1 + y^{tK}} \frac{10^{-3}}{10^{17} f(m_{\bar{q}}, m_{\bar{q}}, m_{\bar{w}}) + f(m_{\bar{q}}, m_{\bar{l}}, m_{\bar{w}})} \right|^2 \text{ yrs}, \quad (4.14)$$

where we have introduced three more unknown parameters: the hadron matrix element parameter β , which ranges from 0.003 to 0.03 GeV^3 ; the ratio of the vacuum expectation values of two Higgs doublets, $\tan \beta_H$; and the parameter y^{tK} , which represents the ratio of the contribution of the third generation relative to the second generation to proton decay. To allow an M_Φ as low as possible, we take $\beta = 0.003 \text{ GeV}^3$, $\sin 2\beta_H = 1$ and $|1 + y^{tK}| = 1$. The experimental limit for this mode is $\tau(p \rightarrow K^+ \bar{\nu}_\mu) > 1.0 \times 10^{32} \text{ yr}$ [21], and this translates into the following lower bound on M_Φ :

$$M_\Phi > 1.2 \times 10^{20} (f(m_{\bar{q}}, m_{\bar{q}}, m_{\bar{w}}) + f(m_{\bar{q}}, m_{\bar{l}}, m_{\bar{w}})) = M_\Phi^{\text{min}}. \quad (4.15)$$

The functions $f(m_1, m_2, m_{\bar{w}})$ come from the dressing of the dimension-five operator with the wino exchange, needed to convert it into suitable four-fermion operators for proton decay [34]. With the parametrization we have adopted for susy masses, the combination that appear in (4.15) is given by:

$$f(m_{\tilde{q}}, m_{\tilde{q}}, m_{\tilde{w}}) + f(m_{\tilde{q}}, m_{\tilde{l}}, m_{\tilde{w}}) = \frac{1}{6.5m_{1/2}} \left(\frac{\xi_0 + 13.5}{\xi_0 + 6} \ln(\xi_0 + 7) - \frac{\xi_0 + 0.5}{\xi_0 - 0.5} \ln(\xi_0 + 0.5) \right). \quad (4.16)$$

So that, M_Φ^{min} depends on $m_{1/2}$ and ξ_0 , decreasing with both of them.

On the other hand, the upper limit on M_Φ and M_Σ is derived by requiring that the Yukawa couplings involving these fields remain as perturbative couplings below the Planck scale. This leads to the conditions $M_\Phi \leq 2M_V$ and $M_\Sigma \leq 1.8M_V$ [27], which combined with (4.13) give:

$$\begin{aligned} \ln M_\Phi &< \frac{15\pi}{37} \left(\alpha_1^{-1}(m_Z) - \alpha_2^{-1}(m_Z) \right) + \frac{15}{148} \sum_i g'_i(m_i, \mu) \\ &\quad - \frac{5}{37} \ln M_\Sigma - \frac{191}{80} + \frac{40}{37} \ln 2 = \ln M_\Phi^{max}. \end{aligned} \quad (4.17)$$

The upper limit on M_Φ depends not only on the susy masses but also on M_Σ .

We have already all the ingredients (Eq. (4.12), (4.15) and (4.17)) to check if the scenario of perturbative unification is compatible with all the constraints on $\alpha_3^{-1}(m_Z)$, M_Φ and the susy masses. In order to compare, we examine first the results obtained with the step-function approximation. In this case, there is no problem in having $\alpha_3(m_Z)$ within its experimental range and the susy masses below $1 TeV$, as it can be seen in Table III.

Some comments about these data. The bounds on the heavy and light mass parameters are derived from imposing the condition $M_\Phi^{min}(m_{1/2}, \xi_0) \leq M_\Phi^{max}(m_i, M_\Sigma)$. In this way, once we calculated the lower bound on M_Φ from (4.15) with the maximum allowed value for ξ_0 (determined by m_0^{max} and $m_{1/2} = 45 GeV$), this lower bound on M_Φ gives us the maximum allowed value for M_Σ . At the same time, the minimum value of M_Σ gives us, on the one hand, the lower bound on ξ_0 (upper values on $m_{1/2}$ and m_0), and also gives us the upper bound on M_Φ (Eq. (4.17)) and the lower bound on m_0 . These latter bounds are derived taking $m_{1/2} = 45 GeV$ and $m_{\tilde{h}} = m_Z$, and therefore are independent of any other constraints on the susy masses. The same does not occur with the other bounds, which depend on the value of the upper limit for the susy masses being considered (mainly m_0^{max}). (We have also taken $m_t = 91 GeV$ and $m_h = 60 GeV$ to get the values of the Table, except to derive $\alpha_3(m_Z)^{(min)}$).

In principle, we do not have any constraint on M_Σ^{min} , except the requirement that there is no large splitting between heavy masses. As M_Φ and M_V will be around $10^{16} GeV$, we have taken $M_\Sigma^{min} = 10^{13} GeV$ to give the results quoted in Table III. With this choice, we see that when imposing the naturalness bound on the susy masses, we get $m_{1/2} < m_Z$, near its lower experimental limit, and also narrow ranges for m_0 , M_Φ and M_Σ : $m_0 \approx 1 TeV$, $M_\Phi \approx 10^{16.7} GeV$ and $M_\Sigma \approx 10^{13} GeV$. We can increase the ranges of $m_{1/2}$ and $m_{\tilde{h}}$ if we take a lower value of M_Σ . For example, with $M_\Sigma^{min} = 10^{10} GeV$ we obtain: $45 GeV \leq m_{1/2} \leq 610 GeV$, $415 GeV \leq m_0 \leq 1 TeV$. In any case, if we maintain the naturalness bound of $1 TeV$ for the susy masses, it is *not* possible to maintain all heavy masses of the same order. Since the value of M_Φ is practically fixed by the experimental limits on proton decay, and M_V diminishes with M_Σ , the most favorable situation will be: $M_\Sigma < M_V \lesssim M_\Phi$.

When we calculate with effective couplings, the values of $\alpha_3^{-1}(m_Z)$ are lower than those obtained with the step-function approximation, mainly due to the thresholds of the massive

gauge bosons. An now, to satisfy the constraints on $\alpha_3(m_Z)$ and M_Φ we need to have m_0 , $m_{\tilde{h}}$ or m_H beyond 1 TeV . The results derived in this case are presented in Table IV. We see that the lower ξ_0 (i.e., the lower squark and slepton masses), the higher will be M_Φ , and therefore higher values of $m_{\tilde{h}}$ or m_H will be needed to get $\alpha_3(m_Z) \leq 0.125$. If squarks and sleptons are below 1 TeV , the Higgs and higgsino masses have to be beyond this bound. Notice that this conclusion is not affected by the value adopted for M_Σ^{min} . It is the value of M_Φ^{min} and the condition $\alpha_3(m_Z) = 0.125$ which fixes the lower bound for $m_{\tilde{h}} = m_H$, independently of M_Σ .

As before, the bounds on the susy masses depend very midly on the values of m_t and m_h , decreasing as m_t and m_h increase. Thus, we have taken the values $m_t = m_h = 200\text{ GeV}$ to give the results in Table IV.

We have seen that, in the context of perturbative gauge coupling unification at one loop order, the use of effective couplings and the requirement of having susy masses not too high, *favor* a value of $\alpha_3(m_Z)$ near its upper experimental bound, and this is independent of whether or not we include the heavy threshold effects. However, when we try to incorporate corrections at two-loop order, or when we take into account heavy threshold effects, the maximum allowed value for $\alpha_3(m_Z)$ is not enough to get unification, and we have to increase the susy masses beyond their naturalness bound. Therefore, when we use effective couplings is not so easy to maintain the naturalness bound for the susy masses, as it was when we treat the thresholds with the step-function approximation.

V. COMPARISON WITH OTHER APPROACHES AND CONCLUSIONS.

In this paper we have studied how threshold effects associated to massive degrees of freedom modify the evolution of gauge coupling constants with scale; we have done it for both the Standard Model and its Minimal Supersymmetric extension. As a explicit application, we have considered the impact of these effects on the perturbative unification of the couplings.

To begin with, we have examined and specified renormalization procedures for computing the threshold functions. In dealing with gauge coupling renormalization, we can distinguish two classes of contributions: those coming from gauge boson vacuum polarization, which are well and uniquely defined, and contributions coming from vertex and external legs, which not so clearly defined. In order to get the latter contribution, we have required that the procedure respects the Slavnov–Taylor identities. This will guarantee the universality of the renormalized coupling and, at the same time, will constrain to some extent the class of particle masses that can contribute to the vertex function Γ : only gauge boson masses. All other massive matter fields contribute to the effective couplings only through the transverse component of the vacuum polarization.

For the QCD gauge coupling, since gluons are massless, the vertex function is simply given by the standard term calculated using an MISP. For the $SU(2)_L \times U(1)_Y$ gauge couplings, the symmetry is broken at low energies, and we have massive gauge bosons. To get the explicit expressions for the vertex function, we follow the arguments given in Ref. [17]. In this way, at the same time we get universal *and* mass dependent couplings, so that in a single stroke we solve another problem related to the electroweak symmetry breaking: the misdiagonalization of the neutral mass matrix.

We remark that we follow Ref. [17] *only* in order to define the vertex function Γ . Our effective couplings differ from their definitions. They define the effective couplings including only vector boson self-energies, that contain at least, one exactly-conserved matter current from the unbroken subgroup $SU(3)_c \times U(1)_{em}$. This condition imposes severe constraints on the kind of matter fields appearing in the effective couplings [11]: there are no contribution from neutral and colorless matter fields. Therefore, any relation of the couplings relevant for gauge coupling unification will be independent, for example, of the neutral higgses and higgsinos. When we study unification with this choice of the effective couplings, we are interpolating between the unbroken $SU(3)_c \times U(1)_{em}$ and the unification group G , but forgetting about the electroweak symmetry breaking in the matter sector of the model.

One can understand the definition of the $g'(q^2)$ and $g(q^2)$ given in Ref. [17] as a direct splitting of the electromagnetic coupling $e(q^2)$, such as:

$$\begin{aligned} \frac{1}{e^2(q^2)} &= \frac{1}{e^2} + \Pi'_{AA}(q^2) \\ &= \left(\frac{1}{g'^2} + \frac{1}{g^2} \right) + (\Pi'_{BA}(q^2) + \Pi'_{W^3A}(q^2)) \\ &= \frac{1}{g'^2(q^2)} + \frac{1}{g^2(q^2)}, \end{aligned} \quad (5.1)$$

where the Π'_{ii} 's include the transverse vacuum polarization and the appropriate vertex function. We choose, instead, to define independently the couplings $e^2(q^2)$ and $g^2(q^2)$. Thus, the relation (5.1) only fixes the coupling $g'^2(q^2)$. With our choice, *all* the particles that interact under $SU(2)_L$ are present, and affect the evolution of the effective couplings. Of course, the high energy limit of the effective coupling is the same for both definitions, as it is dictated by the RGE and the decoupling theorem.

In some sense, our approach is more closely related to the work of Ref. [12] on light threshold effects. These authors related the corrections (δg_i^2) to the low energy gauge couplings due to light thresholds (new physics different from the Standard Model) to experimental quantities measured in scattering processes. Thus, they define independently δg^2 and $\delta(g'^2 + g^2)$, the first being related to the transverse vacuum polarization of the W^\pm , as we have done.

Our work agree with the one presented in Ref. [12] on the importance of light and heavy thresholds for the predictions derived from perturbative unification. In spite of this, the philosophy of our approach is in some sense different. Taking as unifying group the supersymmetric extension of $SU(5)$, they extrapolate the relation given at the unification scale, down to the low energy scale m_Z as

$$12\alpha_2^{-1} - 5\alpha_1^{-1} - 7\alpha_3^{-1} |_{m_Z} = 0; \quad (5.2)$$

We only get this relation when using a *mass independent* renormalization procedure. The experimental values of the couplings are extracted on the bases of the validity of the Standard Model, but this relationship is obtained when we assume an extended model. Therefore, if one wants to make use of the experimental data on the couplings, it is necessary to include the corrections due to the new degrees of freedom¹⁰. This is also the main issue of some

¹⁰We would like to rectify the comment on the results of Ref. [12] that we included in the *Note*

recent work [13]. In these papers, a calculation of complete susy contributions is carried out in order to determine the value of $\sin^2 \theta_W(m_Z) |_{\overline{MS}}$ from experimental quantities, but in the context of the MSSM. These threshold contributions evaluated *at* the scale m_Z tend to decrease the value of $\sin^2 \theta_W(m_Z)$, and as a consequence there is an increase in the predicted value of $\alpha_3(m_Z)$ from unification, hardly compatible with the experimental one. In addition to this, we were concerned on how thresholds effects modified the *evolution* with scale of gauge couplings, to understand how in going from M_X to m_Z these effects will affect the relation (5.2). We have considered a more accurate description of these effects, beyond the leading logarithmic correction given by the step-function approximation. When including the complete threshold function, not only susy thresholds have to be considered, but also any threshold effects due to the massive standard particles. For example, the standard-gauge boson threshold effects, which become relevant in realizing or not a perturbative scenario for the unification of the couplings, as we have shown.

We have found that the predictions derived by imposing perturbative unification of gauge couplings in the MSSM depend on the procedure chosen to treat the thresholds. For example, the effective couplings always favor higher values of $\alpha_3^{-1}(m_Z)$ and susy masses, than the calculation with the step-function. Moreover, although both procedures give good results at one-loop order without including heavy thresholds, when we improve the calculation (2-loop order or include heavy thresholds) the unification with effective couplings *is in conflict* with at *least* one of the constraints we impose on the model: the experimental values for $\alpha_3(m_Z)$ and the proton life-time, or the theoretical naturalness bound for susy masses. In order to maintain the experimental constraints, we would need susy masses beyond the upper bound of 1 *TeV* usually required because of naturalness reasons.

At least, this is the general conclusion within the framework of $SU(5)$ unification, where heavy threshold corrections to the predicted value of $\alpha_3(m_Z)$ are positive. One way to avoid this result would be to considere a different unification group, which gives the reverse sign for this contribution [13]. We would have to look for a cancellation between “light” and “heavy” threshold effects.

added in proof of Ref. [29]. There, we misunderstood the notion of “vertex correction” used in Ref. [12]. Of course, the vertex correction due to new physics (susy masses) are not universal, and do not have to be included, as they correctly did not [35].

APPENDIX A:

In this Appendix we give the analytical expressions for the functions $\Pi_{ii}^T(q^2)$ and $\Gamma(q^2)$ introduced in Section II. These expressions have been calculated using dimensional regularization, and can be written as combinations of the following integrals:

$$B_0(a_1, a_2) = \int_0^1 dx \ln(a_1 x + a_2(1-x) + x(1-x)), \quad (\text{A1})$$

$$B_3(a_1, a_2) = \int_0^1 dx x(1-x) \ln(a_1 x + a_2(1-x) + x(1-x)), \quad (\text{A2})$$

$$C_0(a_1, a_2, \xi a_2) = \int_0^1 dx \int_0^x dy \ln(a_1(1-x) + a_2(x-y+\xi y) + x(1-x)), \quad (\text{A3})$$

$$D_1(a_1, a_2, \xi a_2) = \int_0^1 dx \int_0^x dy \frac{1-x}{(a_1(1-x) + a_2(x-y+\xi y) + x(1-x))}, \quad (\text{A4})$$

$$D_3(a_1, a_2, \xi a_2) = \int_0^1 dx \int_0^x dy \frac{x(1-x)}{(a_1(1-x) + a_2(x-y+\xi y) + x(1-x))}, \quad (\text{A5})$$

$$(1-\xi)H(a_1, a_2) = \frac{1}{2a_2} \int_0^1 dx \int_0^x dy \ln \frac{a_1(1-x+\xi(x-y)) + a_2 y + y(1-y)}{a_1(1-x+\xi(x-y)) + \xi a_2 y + y(1-y)}, \quad (\text{A6})$$

and the linear combinations:

$$D_{31}^+ = D_1 + D_3, \quad D_{31}^- = D_1 - D_3. \quad (\text{A7})$$

Here, we have introduced the variable $a_i = \frac{m_i^2}{-q^2}$, where $(-q^2)$ is the euclidean momentum.

The general transverse contributions to the Π_{ii}^T , which depend on the kind of masses running in the loops, are given by (ξ is the gauge parameter):

(a) Gauge Boson + Ghost:

$$\begin{aligned} (4\pi)^2 \Pi^{(G)}(a_1, a_2) = & -\frac{1}{2} \left\{ \left(\frac{13}{3} - \xi \right) \left(\frac{2}{\epsilon} - \ln \frac{-q^2}{\mu^2} \right) + \frac{2}{3} \right. \\ & -2B_0(a_1, a_2) - 10B_3(a_1, a_2) + 2B_3(\xi a_1, \xi a_2) \\ & -(1-\xi) \left(1 + C_0(a_1, a_2, \xi a_2) + C_0(a_2, a_1, \xi a_1) \right) \\ & + (1+a_1)D_{13}^+(a_1, a_2, \xi a_2) + (1+a_2)D_{13}^+(a_2, a_2, \xi a_2) \\ & \left. + (1-\xi)^2 H(a_1, a_2) \right\}. \end{aligned} \quad (\text{A8})$$

(b) Scalar–Gauge Boson:

$$(4\pi)^2 \Pi^{(SG)}(a_1, a_2) = -(1-\xi)a_2 D_{13}^-(a_1, a_2, \xi a_2), \quad (\text{A9})$$

Here m_1^2 is the scalar mass, and m_2^2 the gauge boson mass.

(c) Scalars:

$$(4\pi)^2 \Pi^{(s)}(a_1, a_2) = \frac{1}{3} \left\{ \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} \right) - 3B_0(a_1, a_2) + 12B_3(a_1, a_2) \right\}. \quad (\text{A10})$$

(d) Fermions:

$$(4\pi)^2 \Pi^{(F)}(a_1, a_2) = \frac{4}{3} \left\{ \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} \right) - 6B_3(a_1, a_2) \right\}. \quad (\text{A11})$$

In the expressions above, we have not included the couplings or group factors. The scale “ μ ” is the unit of mass introduced in dimensional regularization, and ε is given, as it is usual in Modified Minimal Subtraction, by $\frac{2}{\varepsilon} = \frac{2}{n-4} - \gamma + \ln 4\pi$.

In each particular case, we get:

$$\begin{aligned} \Pi_{AA}^T(q^2) &= e^2 \left\{ 2\Pi^{(G)}(a_w, a_w) + \Pi^{(sG)}(\xi a_w, a_w) + \Pi^{(s)}(\xi a_w, \xi a_w) \right. \\ &\quad \left. + \sum_f Q_f^2 \Pi^{(F)}(a_f, a_f) \right\}, \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} \Pi_{WW}^T(q^2) &= g^2 \left\{ 2s_w^2 \Pi^{(G)}(0, a_w) + 2c_w^2 \Pi^{(G)}(a_z, a_w) \right. \\ &\quad + \Pi^{(sG)}(a_h, a_w) + s_w^2 \Pi^{(sG)}(\xi a_w, 0) + s_w^4 \Pi^{(sG)}(\xi a_w, a_z) \\ &\quad \left. + \frac{1}{4} \Pi^{(s)}(a_h, \xi a_w) + \frac{1}{4} \Pi^{(s)}(\xi a_z, \xi a_w) + \frac{1}{4} \sum_{\text{doublets}} \Pi^{(F)}(a_{f_1}, a_{f_2}) \right\}, \end{aligned} \quad (\text{A13})$$

$$\Pi_{gg}^T(q^2) = g_3^2 \left\{ 3\Pi^{(G)}(0, 0) + \frac{1}{2} \sum_{\text{quarks}} \Pi^{(F)}(a_f, a_f) \right\} \quad (\text{A14})$$

In order to obtain the function $\Gamma(q^2) = -(g^2 + g'^2) \Pi_{ZA}^L(q^2) / (gg') m_z^2$, we need the longitudinal term of the diagrams in Fig. (9). The diagrams (9.a) and (9.b) are related and we do not need to calculate both explicitly; the same kind of diagrams contribute to the longitudinal vacuum polarization of the photon, which we know to be zero. Therefore, if we name $A(a_w)$ the longitudinal term of (9.a), and $B(a_w)$ the corresponding to (9.b), they must verify the following identity,

$$\Pi_{AA}^L(q^2) = e^2 m_w^2 (A(a_w) + B(a_w)) = 0 \rightarrow A(a_w) = -B(a_w). \quad (\text{A15})$$

So that, for $\Pi_{ZA}^L(q^2)$ we obtain:

$$\Pi_{ZA}^L(q^2) = -e \frac{g^2}{\sqrt{g^2 + g'^2}} m_w^2 A(a_w) + eg \frac{g'^2}{g^2 + g'^2} m_z m_w B(a_w) = gg' m_w^2 B(a_w), \quad (\text{A16})$$

where,

$$\begin{aligned} B(a_w) &= \left\{ \frac{3+\xi}{2} \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} \right) - 2B_0(\xi a_w, a_w) \right. \\ &\quad \left. + (1-\xi)C_0(\xi a_w, a_w, \xi a_w) - 2(1-\xi)D_{13}^-(\xi a_w, a_w, \xi a_w) \right\}, \end{aligned} \quad (\text{A17})$$

And finally, we get for $\Gamma(q^2)$:

$$\Gamma(q^2) = (g^2 + g'^2) \frac{m_W^2}{m_Z^2} B(a_W) = -g^2 B(a_W). \quad (\text{A18})$$

As we did in the main body of the paper, on the following we redefine $\Pi_{ii}^T \equiv g_i^2 \Pi_{ii}^T$ and $\Gamma \equiv g^2 \Gamma$.

Now, we write the expressions for $\Pi_{ii}^T(q^2) + 2\Gamma_i(q^2)$ in the Landau gauge, and separate in each term the divergent part:

$$\begin{aligned} (4\pi)^2(\Pi_{AA}^T + 2\Gamma(q^2)) = & \\ & -\frac{13}{3} \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + F_G(a_W, a_W) \right) - 3 \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + F_\Gamma(a_W) \right) - F_{SG}(0, a_W) \\ & + \frac{1}{3} \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + F_S(0, 0) \right) + \frac{4}{3} \sum_f Q_f^2 \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + F_F(a_f, a_f) \right), \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} (4\pi)^2(\Pi_{WW}^T + 2\Gamma(q^2)) = & \\ & -\frac{13}{3} \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + s_W^2 F_G(0, a_W) + c_W^2 F_G(a_Z, a_W) \right) - 3 \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + F_\Gamma(a_W) \right) \\ & - F_{SG}(a_h, a_W) - s_W^2 F_{SG}(0, 0) - s_W^4 F_{SG}(a_Z, 0) + \frac{1}{3} \sum_{doublets} \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + F_F(a_{f1}, a_{f2}) \right) \\ & + \frac{1}{6} \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + \frac{1}{2} F_S(a_h, 0) + \frac{1}{2} F_S(0, 0) \right), \end{aligned} \quad (\text{A20})$$

$$\begin{aligned} (4\pi)^2(\Pi_{gg}^T + 2\Gamma_3(q^2)) = & \\ & -11 \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + Cte \right) + \frac{2}{3} \sum_{quarks} \left(\frac{2}{\varepsilon} - \ln \frac{-q^2}{\mu^2} + F_F(a_f, a_f) \right). \end{aligned} \quad (\text{A21})$$

The functions $F_j(a_i)$ contain the threshold effects associated to massive degrees of freedom. Each of these functions has the property that they tend to a constant in the limit where the masses can be neglected as compared to the scale, and they behave as $\ln a_i + O(1/a_i)$ in the limit of heavy masses compared with the momentum scale; except the function $F_{SG}(a_i, a_j)$, which tends to zero in both limits. Having in mind these limits, we can study the behavior of the effective couplings,

$$\frac{1}{g_i^2(q^2)} = \frac{1}{g_i^2(q_0^2)} + \left(\Pi_i^T(p^2) + 2\Gamma_i(p^2) \right) \Big|_{p^2=q_0^2}^{p^2=q^2},$$

for several ranges of scale:

(a) Limit $q^2 > q_0^2 \gg m_i^2$:

$$\frac{(4\pi)^2}{e^2(q^2)} = \frac{(4\pi)^2}{e^2(q_0^2)} + \left(-\frac{13}{3} + \frac{4}{3} \sum_f Q_f^2 \right) \ln \frac{q_0^2}{q^2} = \frac{(4\pi)^2}{e^2(q_0^2)} + \frac{11}{3} \ln \frac{q_0^2}{q^2}, \quad (\text{A22})$$

$$\frac{(4\pi)^2}{g^2(q^2)} = \frac{(4\pi)^2}{g^2(q_0^2)} + \left(-\frac{13}{3} + \frac{1}{6} + \frac{1}{3} \sum_{doublets} \right) \ln \frac{q_0^2}{q^2} = \frac{(4\pi)^2}{g^2(q_0^2)} - \frac{19}{6} \ln \frac{q_0^2}{q^2}, \quad (\text{A23})$$

$$\frac{(4\pi)^2}{g_3^2(q^2)} = \frac{(4\pi)^2}{g_3^2(q_0^2)} + \left(-11 + \frac{2}{3} \sum_{quarks} \right) \ln \frac{q_0^2}{q^2} = \frac{(4\pi)^2}{g_3^2(q_0^2)} - 7 \ln \frac{q_0^2}{q^2}. \quad (\text{A24})$$

Thus, we recover in this limit (negligible masses) the usual expressions derived by minimal subtraction.

(b) Limit $q_0^2 < q^2 \ll m_i^2$: in this case, the massive degrees of freedom must decouple and not contribute to the effective couplings. When working in the Landau gauge, we have to pay special attention to the contribution of the Goldstone bosons and ghosts. In the Landau gauge, we treat these degrees of freedom as massless, and we could argue that the limit of heavy gauge masses will not affect them. If we proceed in this way, we will get the non-decoupling effects proportional to the gauge masses. The correct result is obtained reverting the limits: first, we take the limit $m_i^2/q^2 \rightarrow \infty$ in a general gauge, and then we take $\xi \rightarrow 0$. We will assume this procedure whenever we take the limit of heavy masses¹¹.

(c) Limit $q_0^2 \ll m_i^2 \ll q^2$: In this case we recover, in part, the results obtained by using the step-function approximation. For example, for the electromagnetic coupling we get:

$$\begin{aligned} \frac{(4\pi)^2}{e^2(q^2)} &= \frac{(4\pi)^2}{e^2(q_0^2)} + \lim_{m^2/q^2 \rightarrow 0} \left(\Pi_i^T(q^2) + 2\Gamma_i(q^2) \right) - \lim_{m^2/q_0^2 \rightarrow \infty} \left(\Pi_i^T(q_0^2) + 2\Gamma_i(q_0^2) \right) \\ &= \frac{(4\pi)^2}{e^2(q_0^2)} - 7 \left(\ln \frac{m_W^2}{q^2} + \frac{17}{14} - \frac{2}{21} \right) + \frac{4}{3} \sum_f Q_f^2 \left(\ln \frac{m_f^2}{q^2} + \frac{5}{3} \right) \\ &= \frac{(4\pi)^2}{e^2(q_0^2)} - 7 \ln c_W \frac{m_W^2}{q^2} + \frac{2}{3} + \frac{4}{3} \sum_f Q_f^2 \ln c_f \frac{m_f^2}{q^2}. \end{aligned} \quad (\text{A25})$$

In the last line, we simply have absorbed the constant terms in the logarithms, except for the factor “2/3”, which has a different origin¹². The differences between this expression and the one obtained by the step-function method are in these constants. Their origin is better understood if we study the β function directly, which is given by:

$$\begin{aligned} (4\pi)^2 \frac{\beta_e(q^2)}{e^4(q^2)} &= -\frac{d}{d \ln q^2} \left(\Pi_i^T(q^2) + 2\Gamma_i(q^2) \right) \\ &= -\frac{13}{3} f_g(a_W) - 3f_\Gamma(a_W) + \frac{1}{3} f_s(0, 0) + \frac{4}{3} \sum_f Q_f^2 f_f(a_f), \end{aligned} \quad (\text{A26})$$

where we have defined,

$$\frac{13}{3} f_g(a_W) = \frac{13}{3} \left(1 - \frac{dF_G(a_W, a_W)}{d \ln q^2} \right) - \frac{dF_{SG}(0, a_W)}{d \ln q^2}, \quad (\text{A27})$$

$$f_\Gamma(a_W) = 1 - \frac{dF_\Gamma(a_W)}{d \ln q^2}, \quad (\text{A28})$$

$$f_f(a_f) = 1 - \frac{dF_F(a_f, a_f)}{d \ln q^2}, \quad f_s(0, 0) = 1 - \frac{dF_s(0, 0)}{d \ln q^2}. \quad (\text{A29})$$

¹¹For example, we have considered this when we talk before about the limits of the function $F_{SG}(a_i, a_j)$.

¹²It is a typical threshold effect of the massive gauge bosons, produced by the regularization method employed. Indeed, it is not present when using dimensional reduction [36].

We can see in Fig. (10), where we have plotted $f_f(a_f)$, that the behavior of the functions $f_i(a_j)$ is similar to the step-function. When we integrate β_e from q_0^2 to q^2 , we can divide the integration range into $[q_0^2, m_i^2]$ and $[m_i^2, q^2]$, and approximate the functions $f_i(a_i)$ in each interval for the corresponding limit ($m_i^2/q_0^2 \rightarrow \infty$, $m_i^2/q^2 \rightarrow 0$). In this way, we reproduce exactly the results of the step-function approximation. Nevertheless, if instead we use the Taylor expansion of the functions $f_i(a_j)$ to first order in a_i (improving the latter approximation), we would get for a fermion of mass m_f ,

$$\begin{aligned} - \int_{q_0^2}^{q^2} f_f(m_f^2/y) d \ln y &\simeq - \int_1^{m_f^2/q_0^2} \frac{1}{c_f^\infty y} d \ln y + \int_1^{m_f^2/q^2} (1 + c_f^0 y) d \ln y \\ &= \frac{q_0^2}{c_f^\infty m_f^2} - \frac{1}{c_f^\infty} + \ln \frac{m_f^2}{q^2} + c_f^0 \frac{m_f^2}{q^2} - c_f^0 = \ln c \frac{m_f^2}{q^2}, \end{aligned} \quad (\text{A30})$$

where once again a constant appears in the logarithm. But, neither of the two approximations, step-function or Taylor expansion, is valid, because when we integrate we cross the region $m_f^2/q^2 \approx 1$, where none of these approximations is defined. Due to this, the best procedure is to approximate the complete function, $f_f(a_f)$, by an expression valid for the full energy range, as it is [37]

$$f_f(a_f) \simeq \frac{1}{1 + c_f a_f}. \quad (\text{A31})$$

When we integrate with this function, we get:

$$- \int_{q_0^2}^{q^2} f_f(m_f^2/y) d \ln y \simeq - \int_{m_f^2/q^2}^{m_f^2/q_0^2} \frac{1}{1 + c_f y} d \ln y = \ln \frac{q_0^2 + c_f m_f^2}{q^2 + c_f m_f^2}. \quad (\text{A32})$$

Therefore, we have seen that, for any massive particle, its contribution to the effective coupling can be approximated by a logarithmic function of the form:

$$L_k(q^2, m^2) = \ln \frac{q_0^2 + c_k m^2}{q^2 + c_k m^2}. \quad (\text{A33})$$

With this function, we recover easily the limits (a) and (b) for the effective couplings and, on the other hand, we get a more precise behavior of the thresholds for intermediate scales. The value of the constant c_k depends on the kind of particles we treat (fermion, scalar,...), and if it is or not degenerated with its partner in the loop. In particular for the Standard Model, with or without supersymmetry, and the range of energies of interest, the contributions from loops of non-degenerate particles can be treated as completely degenerate, or that one of the masses can be neglected with respect to the other. A numerical study shows that the best values of the constant c_k for these cases, are those given in Table II.

In the following, and to close this Appendix, we include the analytical expressions of the functions $F_j(a_1, a_2)$:

$$\begin{aligned} F_F(a_1, a_2) &= \frac{5}{3} - \frac{1}{2} \ln a_1 a_2 - 2(a_1 + a_2) - 2(a_1 - a_2)^2 \\ &\quad - \frac{1}{2}(a_1 - a_2)(3(a_1 + a_2) + 2(a_1 - a_2)^2) \ln \frac{a_1}{a_2} \end{aligned}$$

$$+\frac{1}{2}(1-a_1-a_2-2(a_1-a_2)^2)R_{12}L_{12}, \quad (\text{A34})$$

$$\begin{aligned} F_S(a_1, a_2) &= \frac{8}{3} - \frac{1}{2} \ln a_1 a_2 + 4(a_1 + a_2) + 4(a_1 - a_2)^2 \\ &+ \frac{1}{2}(a_1 - a_2)(3 + 6(a_1 + a_2) + 4(a_1 - a_2)^2) \ln \frac{a_1}{a_2} \\ &+ \frac{1}{2}(1 + 2(a_1 + a_2) + 4(a_1 - a_2)^2)R_{12}L_{12}, \end{aligned} \quad (\text{A35})$$

$$\begin{aligned} F_{SG}(a_1, a_2) &= \frac{1}{6}(a_2 - 2a_2^2 + 4a_2a_1 \\ &+ (1 + 3a_1 + 3a_1(a_1 - a_2) + (a_1 - a_2)^3) \ln \frac{a_2}{a_1} \\ &- (1 + 2a_1 - a_2 + (a_1 - a_2)^2)R_{12}L_{12} - 2(1 + a_1)^3 \ln \frac{1 + a_1}{a_1}), \end{aligned} \quad (\text{A36})$$

$$F_R(a) = \frac{5}{3} - \ln a + \frac{a}{3} - \frac{1}{3a} \ln(1 + a) - (1 + a + \frac{a^2}{3}) \ln \frac{1 + a}{a} \quad (\text{A37})$$

$$\begin{aligned} \frac{3}{13}F_G(a_1, a_2) &= A(a_1, a_2) + a_1B(a_1, a_2) + a_2B(a_2, a_1) \\ &+ \frac{C(a_1, a_2)}{a_1} + \frac{C(a_2, a_1)}{a_2} + \frac{a_2D(a_1, a_2)}{a_1} + \frac{a_1D(a_2, a_1)}{a_2} + \frac{E(a_1, a_2)}{a_1a_2}, \end{aligned} \quad (\text{A38})$$

$$\begin{aligned} A(a_1, a_2) &= \frac{121}{18} - \frac{7}{3} \ln a_1 a_2 + 6a_1 a_2 + \frac{13}{3} a_1 a_2 (a_2 - a_1) \ln \frac{a_2}{a_1} \\ &+ (\frac{19}{12} + 3a_1 a_2)R_{12}L_{12}, \end{aligned} \quad (\text{A39})$$

$$B(a_1, a_2) = -4 - \frac{8}{3}a_1 + (\frac{3}{8} - \frac{17}{6}a_1 - \frac{7}{6}a_1^2) \ln \frac{a_1}{a_2} - (\frac{13}{12} + \frac{4}{3}a_1)R_{12}L_{12}, \quad (\text{A40})$$

$$C(a_1, a_2) = -\frac{7}{24} \ln a_1 a_2 + \frac{7}{12} \ln(1 + a_2) + \frac{1}{3}R_{12}L_{12}, \quad (\text{A41})$$

$$\begin{aligned} D(a_1, a_2) &= \frac{1}{24} \left((23 + 17a_2 - 2a_2^2 - 4a_2^3)(2 \ln(1 + a_2) - \ln a_1 a_2) \right. \\ &\left. + (15 + 2a_2 - 4a_2^2)R_{12}L_{12} \right), \end{aligned} \quad (\text{A42})$$

$$E(a_1, a_2) = \frac{1}{24} (\ln a_1 a_2 - 2 \ln(1 + a_1) - 2 \ln(1 + a_2) - R_{12}L_{12}), \quad (\text{A43})$$

where we have introduced:

$$\begin{cases} R_{12} = \sqrt{1 + 2(a_1 + a_2) + (a_1 - a_2)^2}, \\ L_{12} = \ln \left| \frac{1 + a_1 + a_2 - R_{12}}{1 + a_1 + a_2 + R_{12}} \right|. \end{cases} \quad (\text{A44})$$

We note that as regularization procedure we use dimensional regularization to work with the Standard Model, and dimensional reduction to work within the MSSM. Contributions from fermion and scalars are the same in both procedures, the only difference being in the gauge boson contribution. For translating to dimensional reduction, we have only to subtract a constant term “2/3” from equation (A39).

APPENDIX B:

In this appendix we approximate the effective couplings at the 2-loop order. The RGE for gauge couplings at the 2-loop order are given, in general, by [38,7]

$$\frac{d\alpha_i(\mu)}{d\ln\mu} = \frac{b_i(\mu)}{2\pi}\alpha_i^2 + \frac{b_{ij}(\mu)}{8\pi^2}\alpha_j(\mu)\alpha_i^2(\mu). \quad (\text{B1})$$

When we calculate the coefficient b_i and b_{ij} using an MISP, we obtain constant values for b_i and b_{ij} ; but when using an MDSP and we include the dependence on the masses, these coefficients gain a dependence with the scale μ , through the ratios (m_k/μ) . The $b_i(\mu)$ take into account the threshold effects at 1-loop order, that we have calculated in Appendix A. Now, we want evaluate which is the threshold contribution at 2-loop order, included in the $b_{ij}(\mu)$, with respect to that at 1-loop order, without actually performed the explicit calculation.

When we integrate (B1) from m_Z to an arbitrary scale μ , we get:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu) \big|_{1-loop} - \frac{1}{4\pi} \int_{m_Z}^{\mu} \frac{b_{ij}(\mu')}{b_j(\mu')} d\ln\alpha_j(\mu'). \quad (\text{B2})$$

In order to evaluate the contribution of the second term, we choose an intermediate scale, $m_Z < \mu_1 < \mu$, such that $m_i \ll \mu_1$, for all the masses m_i . For example, for the Standard Model and its supersymmetric extension, we have $m_i \leq 1 \text{ TeV}$, and thus it is sufficient if we take $\mu_1 = 10 \text{ TeV}$. In the range $\mu' \geq \mu_1$, the thresholds at 1-loop order and 2-loop order are negligible, and then the coefficients $b_i(\mu')$ and $b_{ij}(\mu')$ remain as constants. Therefore, we can write:

$$\begin{aligned} \alpha_i^{-1}(\mu) &= \alpha_i^{-1}(\mu) \big|_{1-loop} - \frac{1}{4\pi} \int_{m_Z}^{\mu_1} \frac{b_{ij}(\mu')}{b_j(\mu')} d\ln\alpha_j(\mu') - \frac{1}{4\pi} \int_{\mu_1}^{\mu} \frac{b_{ij}}{b_j} d\ln\alpha_j(\mu') \\ &= \alpha_i^{-1}(\mu) \big|_{1-loop} - \frac{1}{4\pi} \int_{m_Z}^{\mu} \frac{b_{ij}}{b_j} d\ln\alpha_j(\mu') - \frac{1}{4\pi} \int_{m_Z}^{\mu_1} \left(\frac{b_{ij}(\mu')}{b_j(\mu')} - \frac{b_{ij}}{b_j} \right) d\ln\alpha_j(\mu') \\ &\equiv \alpha_i^{-1}(\mu) \big|_{1-loop} + \frac{b_{ij}}{4\pi b_j} \ln \frac{\alpha_j(m_Z)}{\alpha_j(\mu)} + \sum_j \Delta_{ij}. \end{aligned} \quad (\text{B3})$$

Now, the 2-loop threshold effects are included in the function Δ_{ij} , and this is the function in which we are interested. To evaluate the order of magnitude of this function, we approximate the integrand in Δ_{ij} by a straight line in the variable $\ln\alpha_j(\mu')$,

$$\left(\frac{b_{ij}(\mu')}{b_j(\mu')} - \frac{b_{ij}}{b_j} \right) \simeq a_{ij} + c_{ij} \ln\alpha_j(\mu'), \quad (\text{B4})$$

with the condition that it passes through the points m_Z and μ_1 , i.e.,

$$\mu' = m_Z \rightarrow a_{ij} + c_{ij} \ln\alpha_j(m_Z) = \left(\frac{b_{ij}(m_Z)}{b_j(m_Z)} - \frac{b_{ij}}{b_j} \right) \equiv B_{ij}(m_Z), \quad (\text{B5})$$

$$\mu' = \mu_1 \rightarrow a_{ij} + c_{ij} \ln\alpha_j(\mu_1) = \left(\frac{b_{ij}(\mu_1)}{b_j(\mu_1)} - \frac{b_{ij}}{b_j} \right) \simeq 0. \quad (\text{B6})$$

With these conditions, the expression for Δ_{ij} is reduced to:

$$\Delta_{ij} = \frac{B_{ij}(m_z)}{8\pi} \ln \frac{\alpha_j(m_z)}{\alpha_j(\mu)}. \quad (\text{B7})$$

The final step consists of taking at 2-loop order the same kind of threshold functions, $f_i(m_i/\mu)$, that at 1-loop order, i.e., a function bounded between 0 and 1. For example, $b_{23} \sim 8 + 4f_{top}$, and then the value of Δ_{23} will be between 0.013 ($f_{top} = 0$) and 0.003 ($f_{top} = 1$). Checking for different values, the corrections to α_i^{-1} , given by $\Delta_i = \sum_j \Delta_{ij}$, are never higher than 0.3%, and therefore, negligible.

Thus, the 2-loop effective couplings can be approximated by:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu) |_{1-loop} + \frac{b_{ij}}{4\pi b_j} \ln \frac{\alpha_j(m_z)}{\alpha_j(\mu)}, \quad (\text{B8})$$

where we only need to include explicitly the threshold functions at 1-loop order.

This approximation is independent of the chosen regularization procedure, in particular if dimensional regularization or dimensional reduction. The regularization procedure does not affect the coefficients of the RGE's for the gauge coupling when we work with \overline{MS} and always preserve its form (at one or two loops). With an MDSP, it easily seen that the function $b_i(\mu)$ at one-loop order does not depend on the regulator. When working at two-loop order, we would have to check how the *explicit* expressions of the $b_{ij}(\mu)$ -coefficients depend on the regularization method. However, here we have only made use of the general properties of these functions, i.e., their behavior in the limiting cases $m/\mu \rightarrow 0$ and $\mu/m \rightarrow 0$. And this behavior is independent of the regularization procedure, as well of the particular mass dependent subtraction procedure one uses.

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FIGURES

FIG. 1. The 1-loop particle contributions to the Π_{gg}^T function. (a) Gluon contributions plus ghost (ω_i) contributions. (b) Fermion contributions.

FIG. 2. Evolution of the three couplings of the SM calculated with the three 1-loop procedures: effective couplings (solid lines), step-function (dashed lines), and \overline{MS} (dotted lines).

FIG. 3. Derivative of the effective coupling $\alpha_2^{-1}(\mu)$ respect to $\ln \mu$ (solid line). The dashed line shows the derivative within the step-function approximation.

FIG. 4. Evolution of the three couplings of the SM at 2-loop order with \overline{MS} (dashed lines), and 1-loop order with a mass dependent method (solid lines). We also plot 1-loop \overline{MS} for comparison (dotted lines).

FIG. 5. Evolution of the three couplings of the MSSM at 1-loop order calculated with step-function (solid lines) and effective couplings (dashed lines), for different values of the susy mass parameters: (a) $m_{1/2} = 45 \text{ GeV}$ and $m_0 = m_Z$, (b) $m_{1/2} = m_0 = 1 \text{ TeV}$; we take $m_{\tilde{h}} = m_+ = m_H = m_0$ and $m_t = m_h = 200 \text{ GeV}$. We include in the plot the experimental error band for each coupling.

FIG. 6. Same as Fig. 5.a, but now without including the threshold effects due to the massive gauge bosons in the evolution of α_2^{-1} .

FIG. 7. Values of $\alpha_3^{-1}(m_Z)$, compatible with the unification condition (Eq. 19), calculated with effective couplings at 1-loop order, for different values of $m_{\tilde{h}} = m_H$: m_Z , 1 TeV , 10 TeV , 100 TeV . Dotted lines are the experimental limits on $\alpha_3^{-1}(m_Z)$ and $m_{1/2}$; solid lines are for $m_t = m_h = 200 \text{ GeV}$, and dashed lines for $m_t = 91 \text{ GeV}$ and $m_h = 60 \text{ GeV}$. The straight lines (solid for $m_t = m_h = 200 \text{ GeV}$ and dashed for $m_t = 91 \text{ GeV}$, $m_h = 60 \text{ GeV}$) are the upper limit obtained for $\alpha_3^{-1}(m_Z)$ when imposing $M_X = 10^{16} \text{ GeV}$ ($m_{\tilde{h}} = m_H$ increase along these lines from bottom to top). The allowed region for $\alpha_3^{-1}(m_Z)$ are to the left of the straight lines, and between the dotted lines, $8 \leq \alpha_3^{-1}(m_Z) \leq 9.2$ and $m_{1/2} \geq 45 \text{ GeV}$.

FIG. 8. Same as Fig. 7, but with $\alpha_3^{-1}(m_Z)$ calculated with the step-function approximation.

FIG. 9. The 1-loop particle contributions to the Π_{ZA}^L function, *i.e.* to Γ' . (a) Gauge boson plus ghost. (b) Scalars and gauge bosons.

FIG. 10. Function $f_f(a_f)$ respect to $-\log_{10}(a_f) = \log_{10}(q^2/m_f^2)$. As it is shown in the plot, the function tends to 1 in the limit of neglecting mass respect to q^2 ($-\log_{10}(a_f) \rightarrow \infty$), while it tends to 0 (decoupling) in the limit of heavy mass ($-\log_{10}(a_f) \rightarrow -\infty$).

TABLES

TABLE I. Experimental values of $\alpha_3(m_Z)$.

Experiment	Central Value	Error
ALEPH jets	0.125	± 0.005
DELPHI jets	0.113	± 0.007
DELPHI (e^+e^-)	0.118	± 0.005
L3 jets	0.125	± 0.009
OPAL jets	0.122	± 0.006
OPAL τ	0.123	± 0.007
J/Ψ	0.108	± 0.005
Deep Inelastic	0.111	± 0.005
UA6	0.112	± 0.009

TABLE II. Fitted values of c_k .

Massive particles	c_k
1 Gauge boson	2
2 Gauge boson	5
1 Fermion	2.5
2 Fermions	5
1 Scalar	4
2 Scalars	10

TABLE III. Values calculated with the step-function approximation^a.

m_0^{max}	ξ_0^{max}	M_Φ^{min}	M_Σ^{max}	α_3^{min}	$m_{1/2}^{max}$	ξ_0^{min}	m_0^{min}	M_Φ^{max}	α_3^{max}
1000	494	$10^{16.58}$	$10^{14.5}$	0.112	91	121	742	$10^{16.79}$	0.125
2000	1975	$10^{16.06}$	$10^{16.3}$	0.108	358	31	742	$10^{16.79}$	0.125

^a Mass values in GeV .

TABLE IV. Values calculated with the effective couplings^a.

m_0^{max}	ξ_0^{max}	m_h^{min}	M_Φ^{min}	M_Σ^{max}	α_3^{min}	$m_{1/2}^{max}$	ξ_0^{min}	M_Φ^{max}	α_3^{max}
1536	1165	1536	$10^{16.26}$	$10^{16.49}$	0.125	45	1165	$10^{16.26}$	0.125
3000	4444	370	$10^{15.75}$	$10^{16.53}$	0.120	239	158	$10^{16.50}$	0.125
4000	7901	198	$10^{15.53}$	$10^{16.54}$	0.118	794	26	$10^{16.61}$	0.125
5000	12348	121	$10^{15.35}$	$10^{16.55}$	0.116	2784	3	$10^{16.69}$	0.125

^a Mass values in GeV .

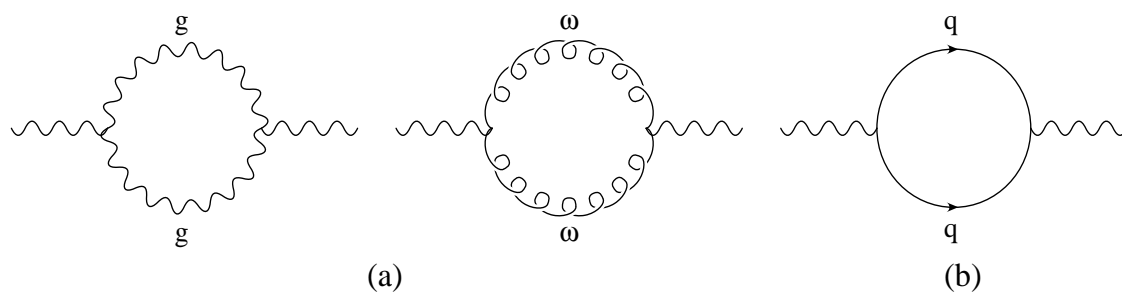


Figure 1

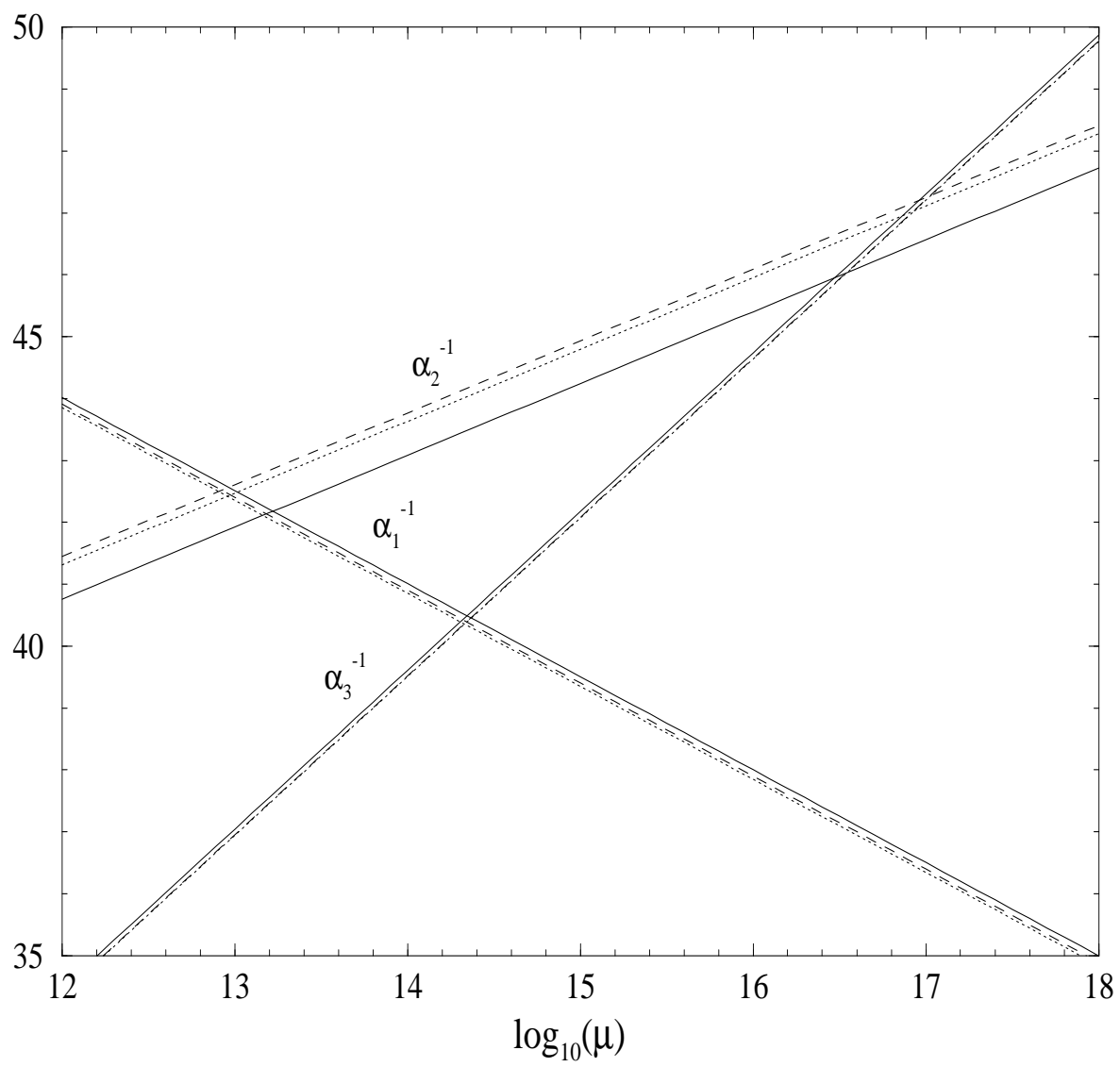


Figure 2

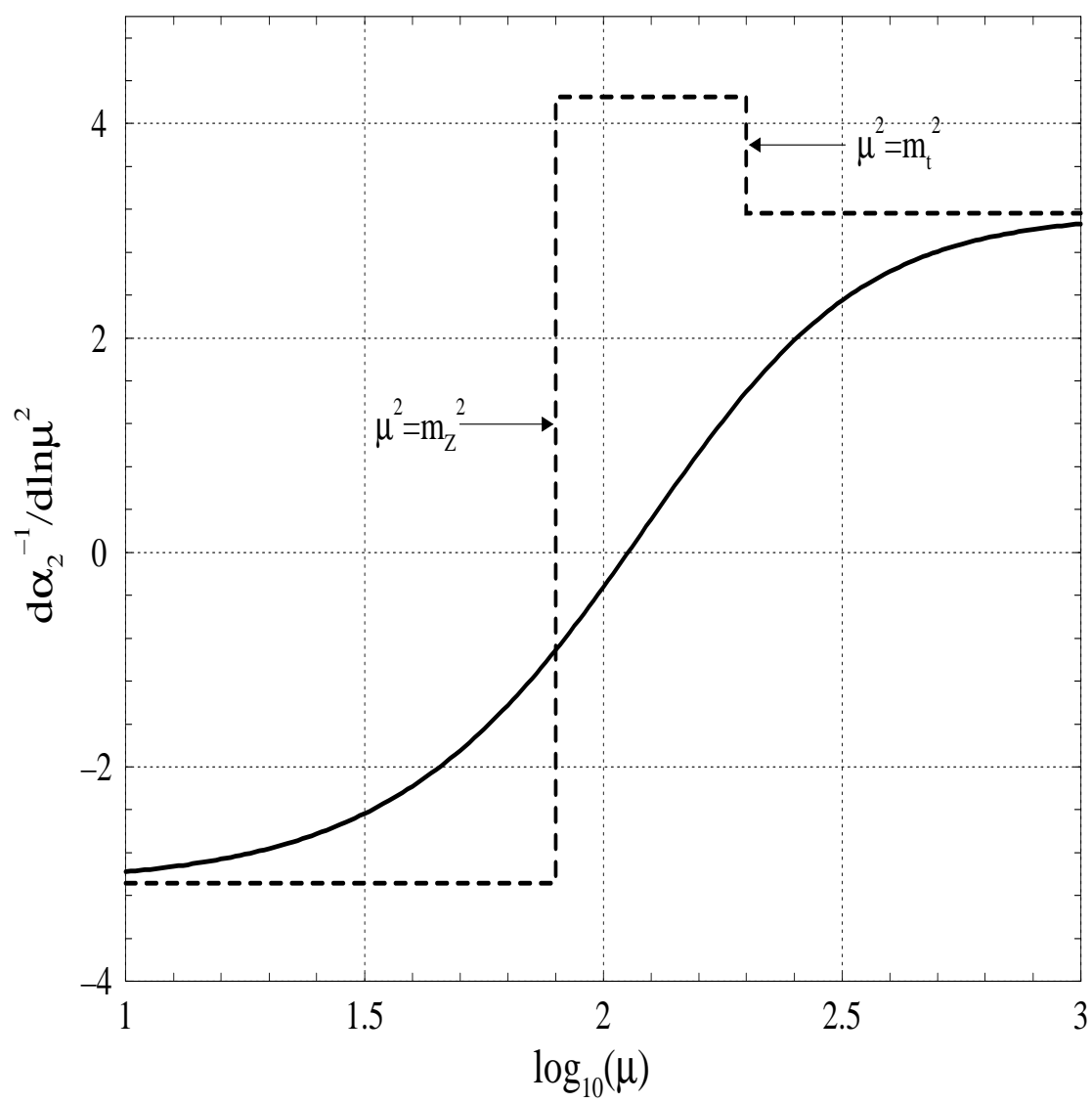


Figure 3

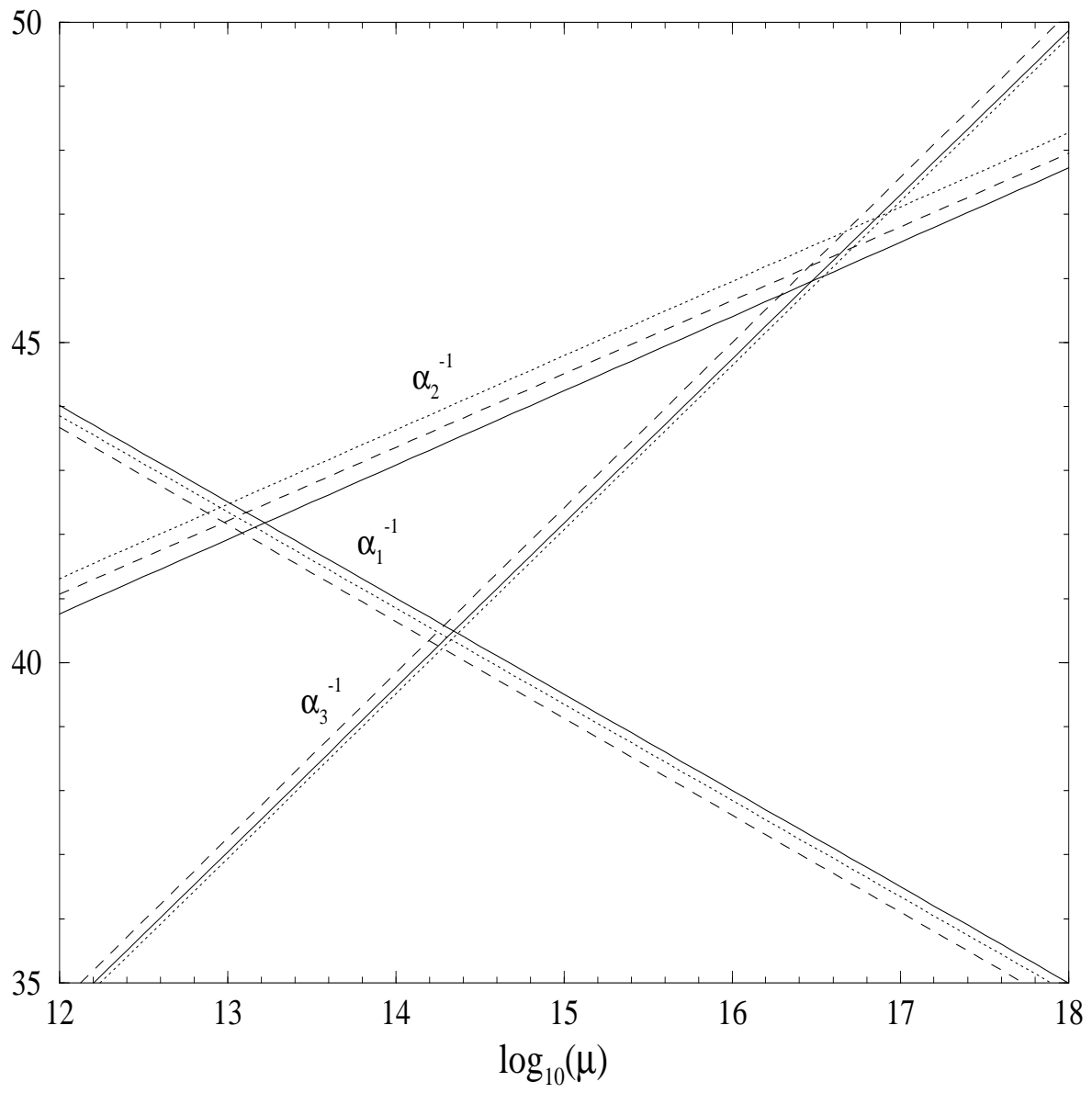


Figure 4

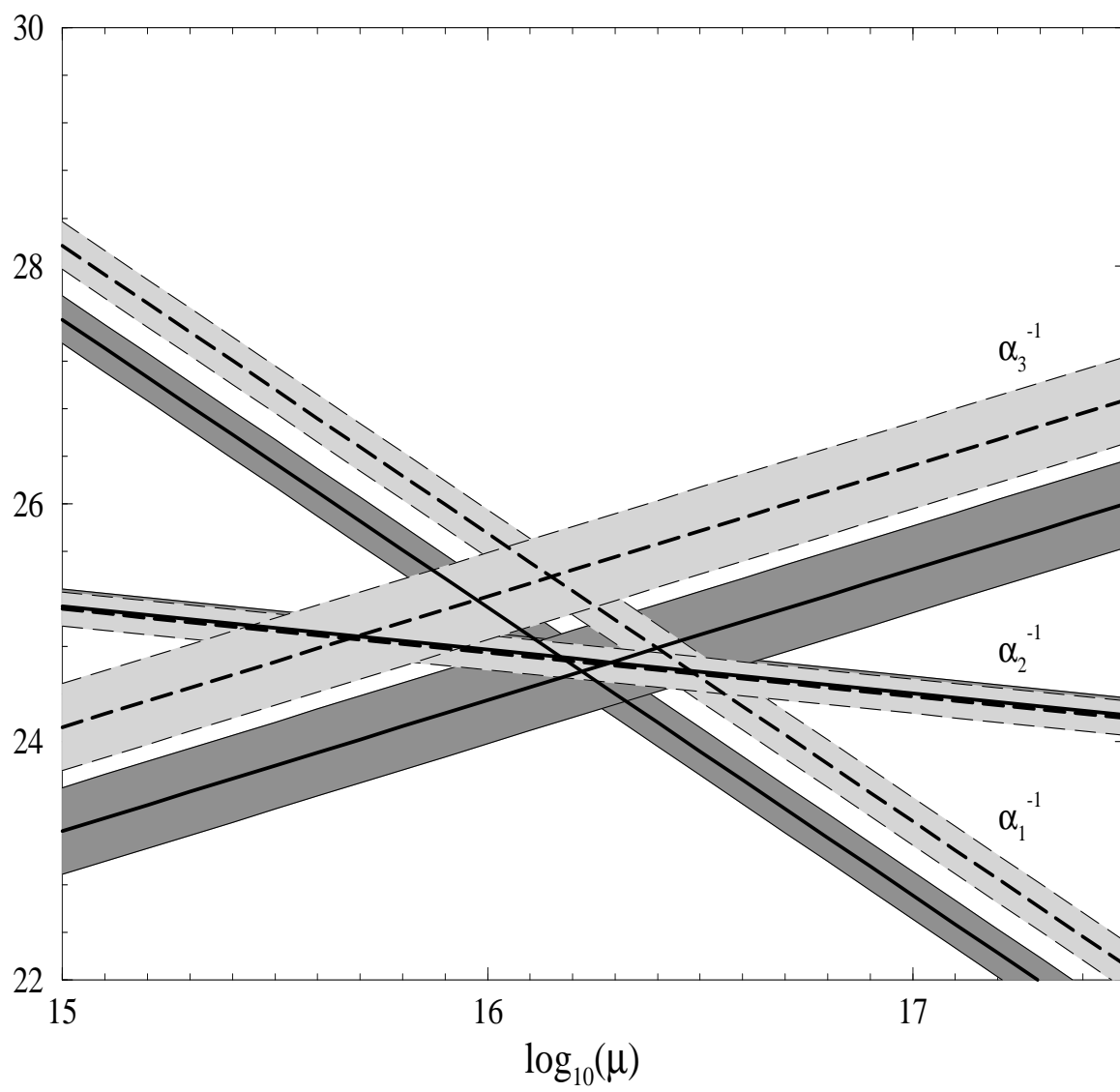


Figure 5.a

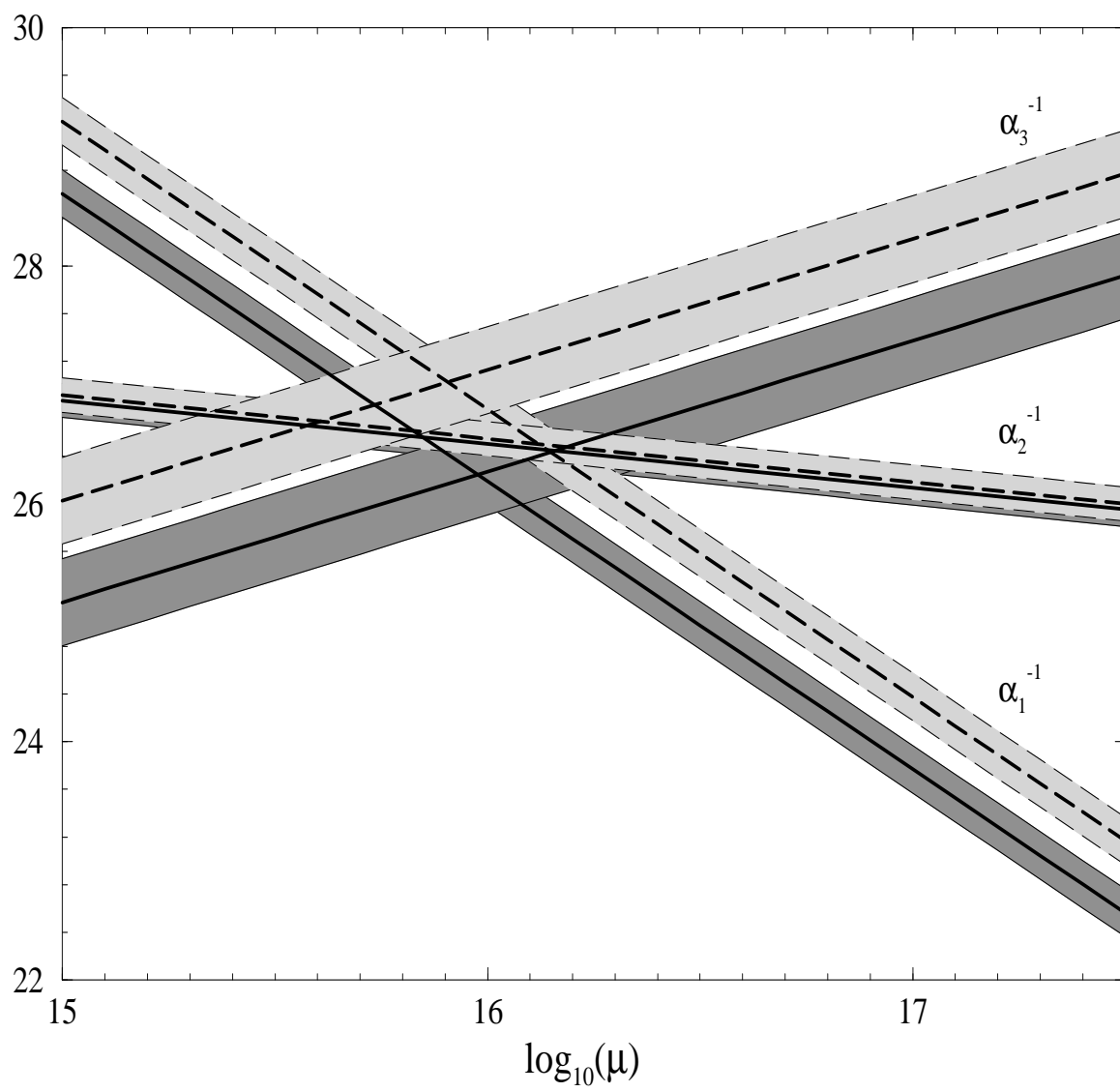


Figure 5.b

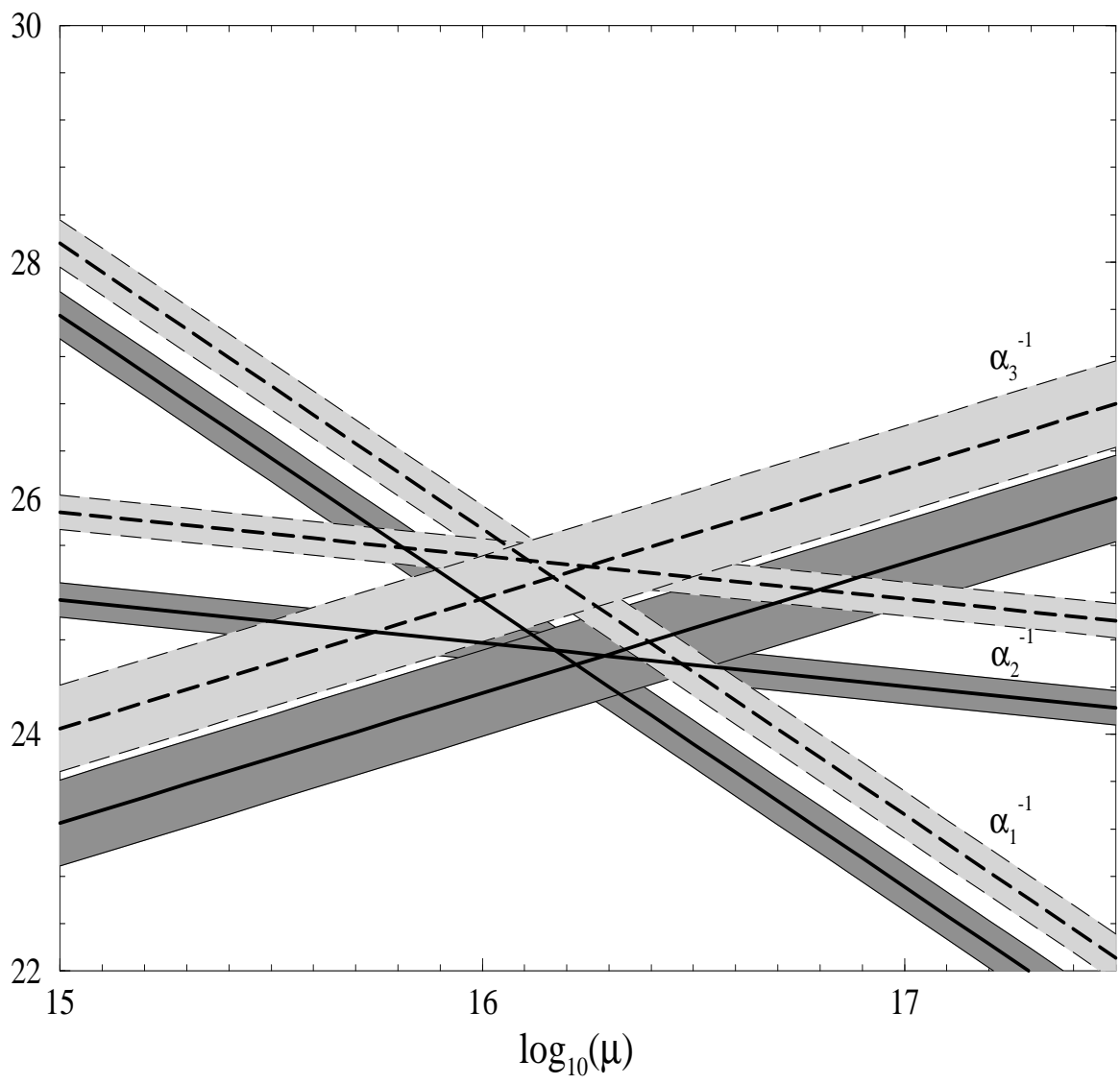


Figure 6

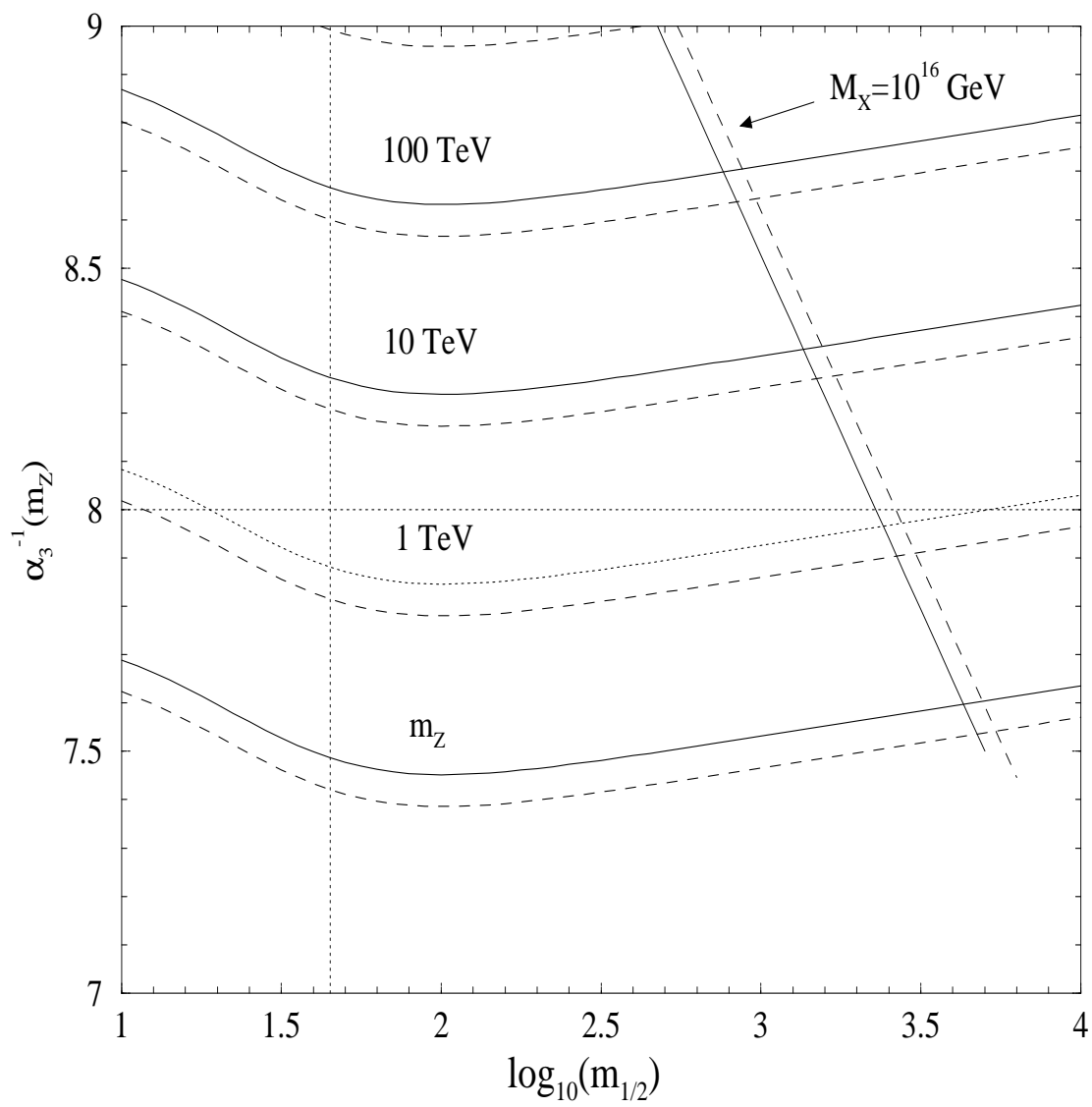


Figure 7

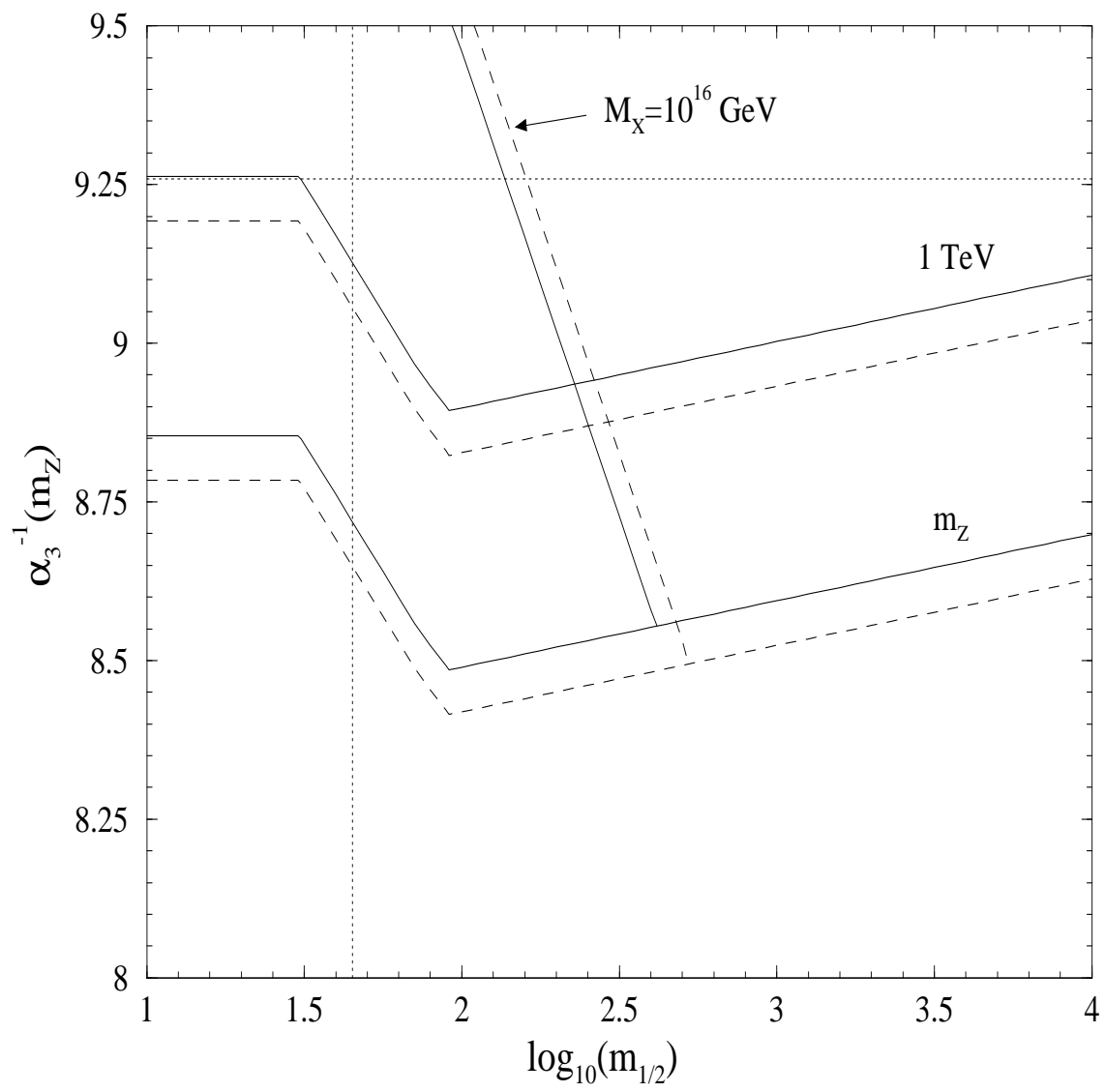


Figure 8

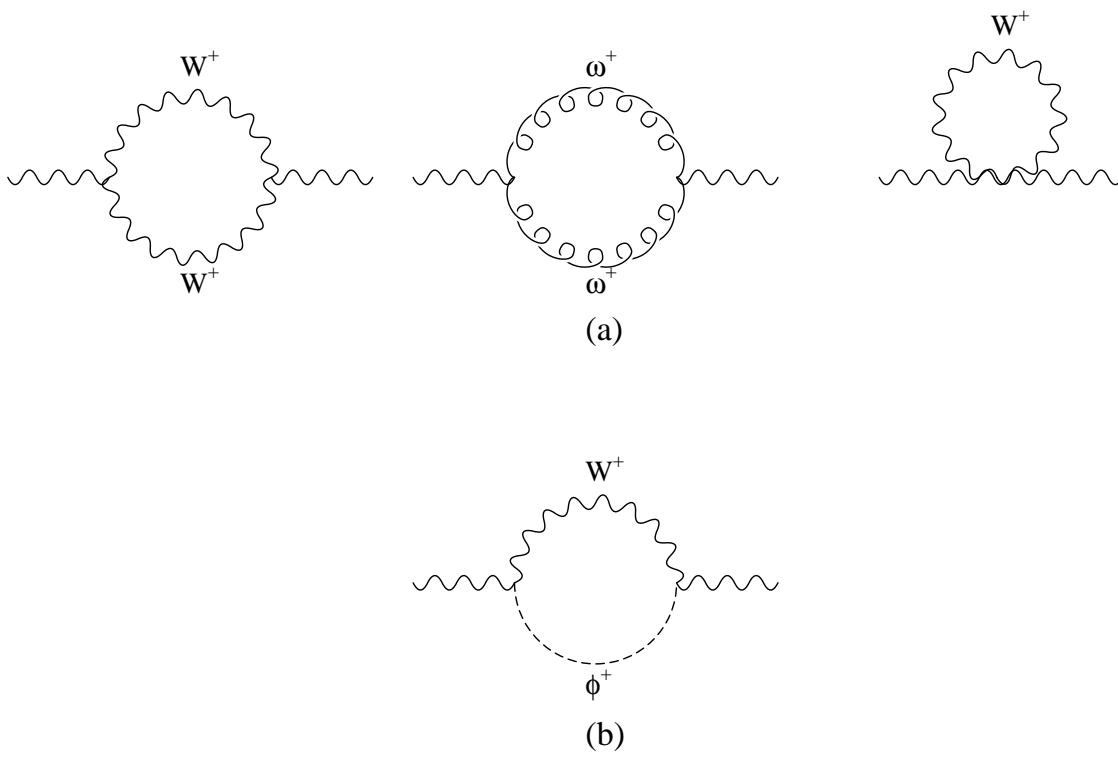


Figure 9

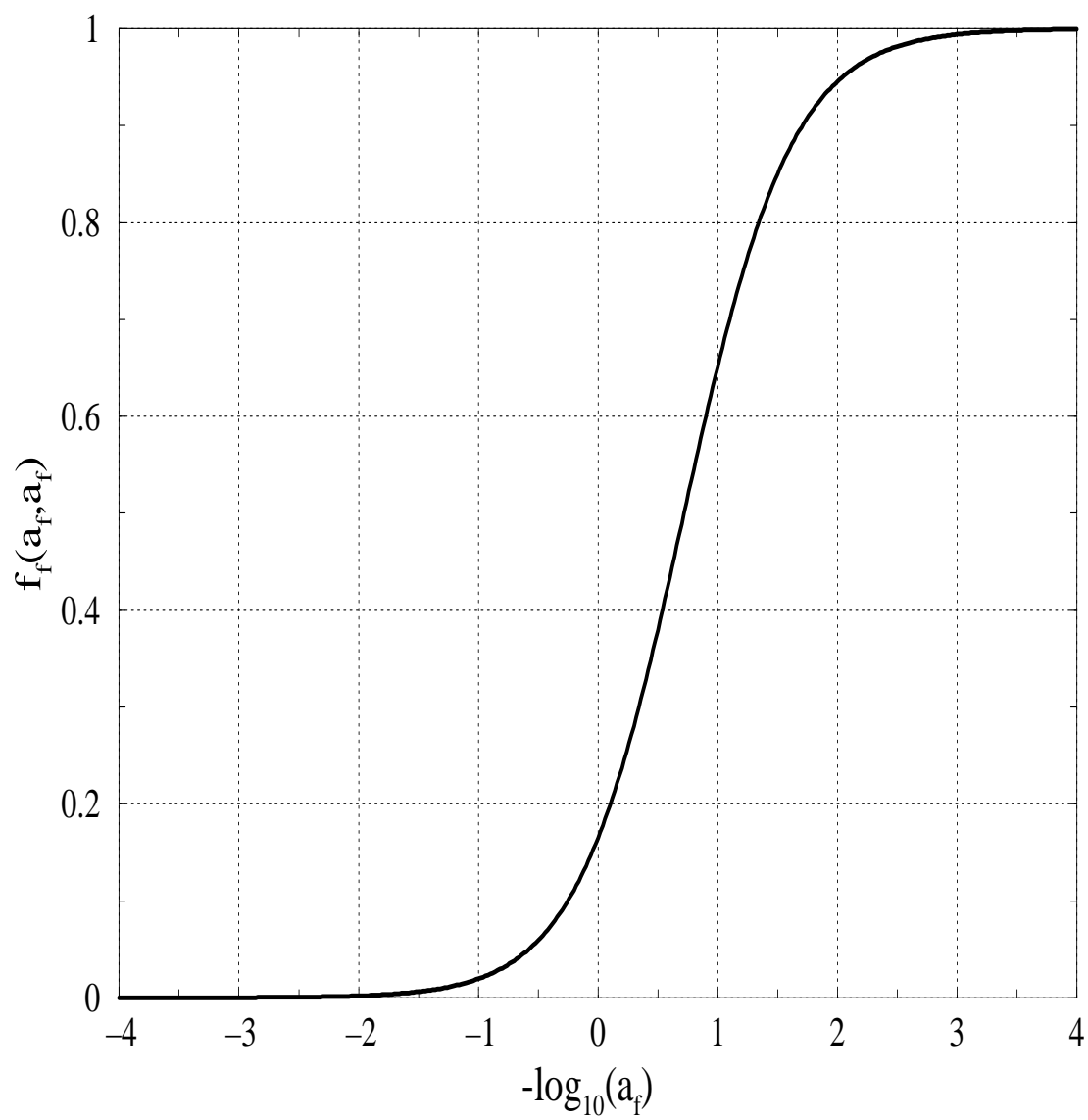


Figure 10